InterTides

maps of the intertidal by interpolation of tidal gauge data

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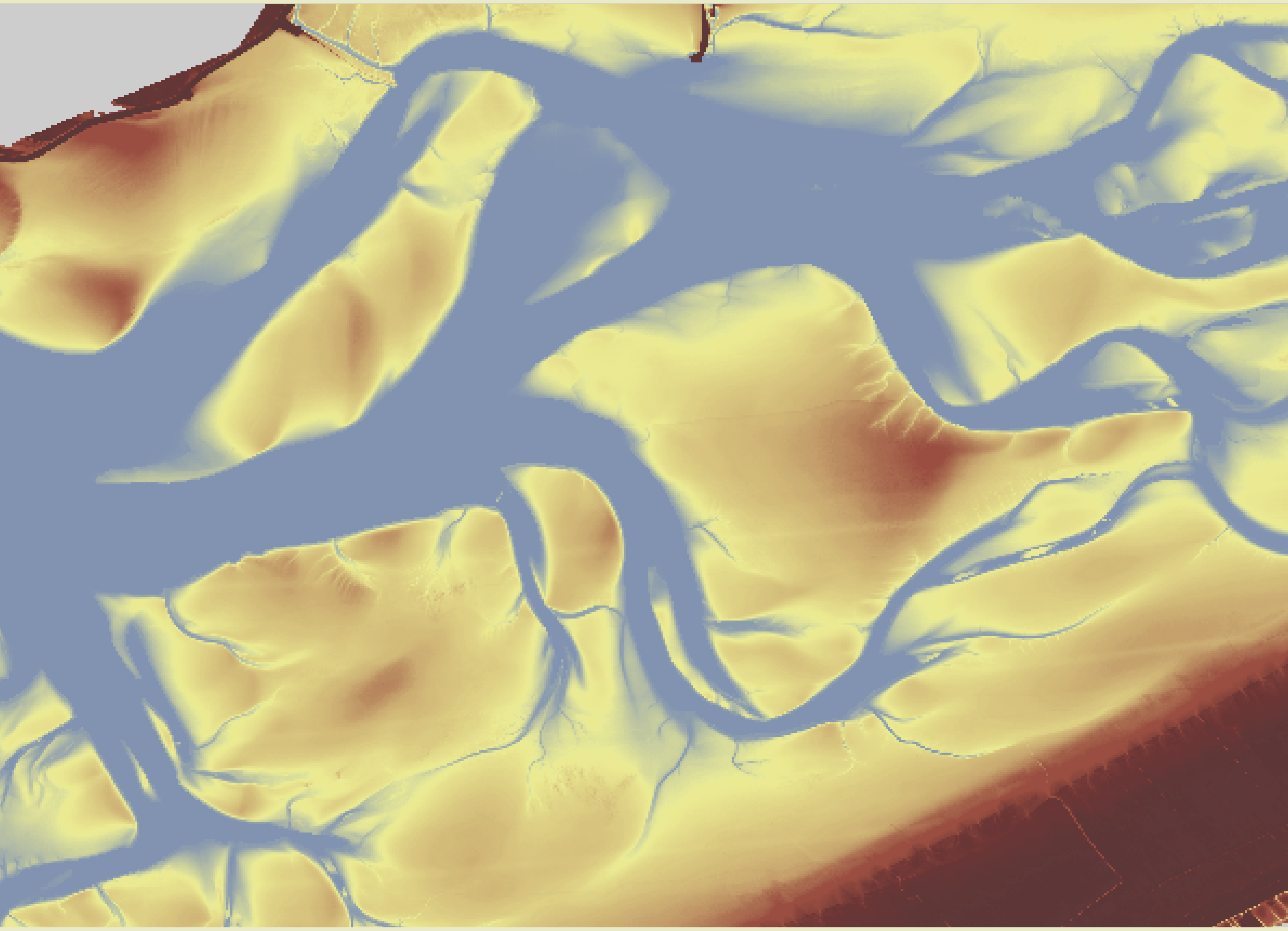

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C. Rappoldt[1], O.R. Roosenschoon[2], D.W.G. van Kraalingen[3]

[1]EcoCurves BV, Kamperfoelieweg 17, 9753 ER Haren, The Netherlands
email: kees.rappoldt@ecocurves.nl

[2]Roosenschoon Webdesign, Akkersdreef 9, 6871 LN Renkum, The Netherlands
email: onno@roosenschoon.info

[3]nAbleWare, Van Eesterenlaan 1, 6708 RL Wageningen, The Netherlands
email: info@nableware.nl

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This report is a concise manual of the program InterTides. This program interpolates between tidal gauge data in order to construct water level maps and exposure time maps for the Wadden Sea. For most map points linear interpolation is used between three nearby measured values. The maps are calculated as ASCII grid files and can be combined in Geographic Information Systems with other data for the area studied. In addition to map construction, the program also calculates distributions of flooding and exposure time for user defined points.

Keywords: Wadden Sea, water level, exposure time

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The most recent version of this manual is available from
www.ecocurves.nl/Support/InterTides/InterTidesManual.pdf

Kamperfoelieweg 17, 9753 ER Haren (gn), Nederland
tel.: (050) 5370979; email: kees.rappoldt@ecocurves.nl

Frontpage: “Part of the Wadden Sea at May, 20 2000 in the afternoon at 16:00”

[EcoCurves September 2014]

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Chapter 1

Introduction to InterTides

InterTides most basic function is interpolation between water levels measured at the tidal gauge stations in and around the Wadden Sea. Based on this functionality one of six different tasks or functions can be selected in the user interface.

An InterTides run requires choosing one of these functions. Depending on this choice, a few required and optional parameters must be set. These parameters deal with the selection of an area, the selection of a time or a period, the choice of a height map, the choice of interpolation settings, and a few output options.

All parameters are listed and discussed in Chapter 2. Here we briefly explain the meaning of "area selection" and then list the various functions.

1.1 Area selection

Most functions of InterTides require the selection of an area. A map with the outlines of the Wadden Sea is part of the InterTides installation. This is the so called "region map". *Outside the region map nothing can be calculated.*

Inside the Wadden Sea region a subarea can be selected from a predefined list. Another possibility is defining a rectangular area by means of the cropping parameters, a lower left edge and a rectangle size.

These two methods of area selection can be combined. If the cropping rectangle selects part of a subarea, only this part of the subarea is used. If the defined rectangle lies around the subarea, maps calculated for the subarea are augmented with NODATA values. Hence, this is a way in which various subareas can be mapped inside the same user-defined rectangle in subsequent InterTides runs.

1.2 Functions of InterTides

1.2.1 Waterlevel map

For the selected area a map is calculated with the water levels at a user-defined time. Hence, the required parameters are the name which is given to the calculated map, the date and time and the cell size. Without further area selection the map is generated for the entire region. An example is the picture in Figure 1.1.

The water level map is generated in the form of an ASCII grid file which can be

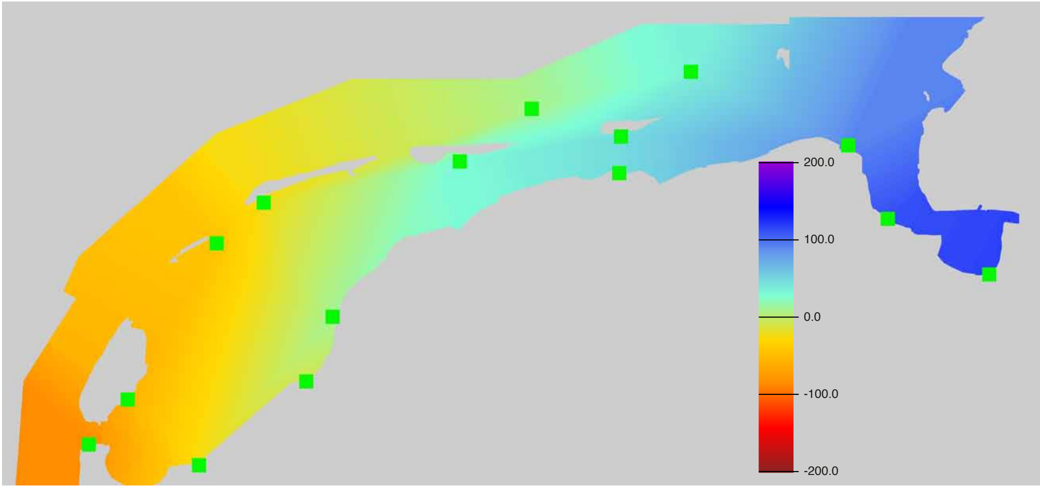

Figure 1.1. Interpolated water levels calculated for May, 20 2001 at 10:00AM. The scale is in cm. The green points mark the positions of the tidal stations.

imported in a GIS program. For direct inspection a TIFF bitmap is written with a colour scale corresponding to the range $[-200, +200]$ in centimetre. This level range is an optional parameter which can be changed in the user interface. Colour scale and associated level range are written to an output file in EPS format (the legend file).

1.2.2 Waterlevel movie

Instead of a single date and time, a begin and end time is specified by the user. InterTides then generates a series of TIFF files similar to Figure 1.1 and these are combined into a movie of the water level. By choosing a smaller or larger time step (in minutes) the changes in water level are presented slowly or more quickly.

A waterlevel movie can be generated for the entire region or for a selected (cropped) subarea. The movie is downloadable in WMV format.

1.2.3 Exposure time map

The calculation of an exposure time requires the use of a height map. A point is considered to be exposed if the estimated water level is below its height on the height map. At first, the selected height map is read at the requested resolution in order to construct a height map for the region or (cropped) subarea.

Then, for each pixel (or cell) on this height map, InterTides calculates the fraction of time the point is exposed. This fraction is found by calculating water levels for all 10 minute intervals inside the selected period, usually spanning several years. The exposed fractions for all points together form a map of the (fractional) exposure time.

An interesting option is the choice of a season, which is the time between two dates (a day and a month). If a season is defined the exposure time map is still calculated for all selected years, but now only for the selected season within those years.

Like a water level map, an exposure time map is generated in the form of an ASCII

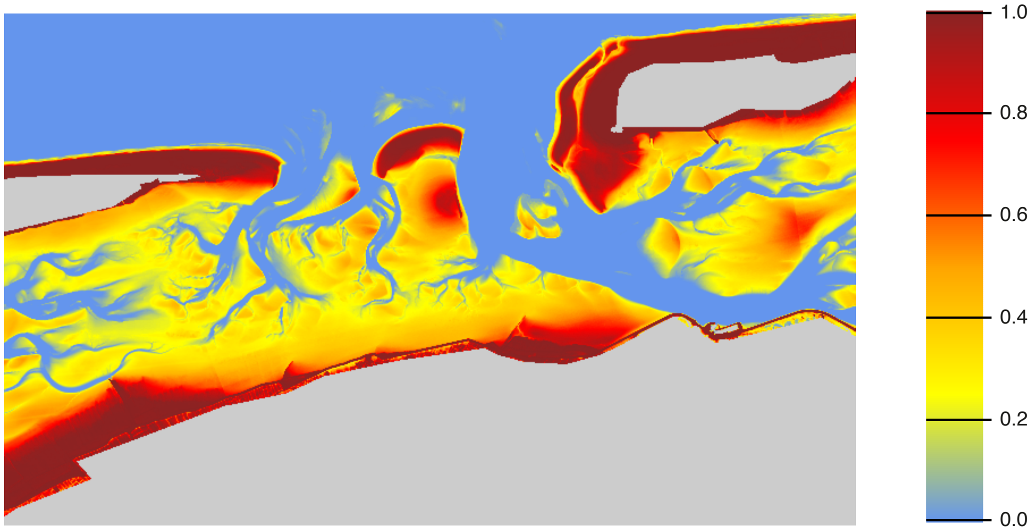

Figure 1.2. Exposure time map for the tidal basins "Pinkegat" and "Zoutkamperlaag" calculated for the period between 01-jan-2001 and 31-dec-2013.

grid file which can be imported in a GIS program. And again, for direct inspection a TIFF bitmap is written with a colour scale corresponding to the 0.0 (never exposed) and 1.0 (always exposed). Optionally the value for "always exposed" can be set to 100, which leads to a map with percentages of time.

Figure 1.2 shows an example map for the area between Ameland and Schiermonnikoog. The map is based on the "cycle 5 height map" of Deltares and is calculated for the period between 01-jan-2001 and 31-dec-2013.

In order to calculate an "honest" fraction of time, the begin and end times of the period can be automatically adjusted to the nearest high tide time at the station which is nearest to the centre of the map. This adjustment is optional and can be switched on in the input screen. In the above example begin and end times were adjusted to the high tide times 1:10 (at 01-Jan-2001) and 20:40 (at 31-Dec-2013) for Wierumergronden.

The synchronisation option for the calculation of Figure 1.2 was set at "M2 phase" (for details see sections 2.4 and 3.4).

1.2.4 Exposure status movie

The exposure status of a point is "flooded" or "exposed". The movie produced by this function is a visualisation of the periodic flooding of the area. For the exposed parts the underlying height map is shown. The flooded parts are blue.

As an example, Figure 1.3 shows the exposure status at 1-sep-2012, 2:12 of the area in Figure 1.2. This is just a single frame of the calculated movie which is based again on the Cycle 5 height map. The legend is the height map legend and the lower heights are clearly not shown since they are flooded at the time and made blue.

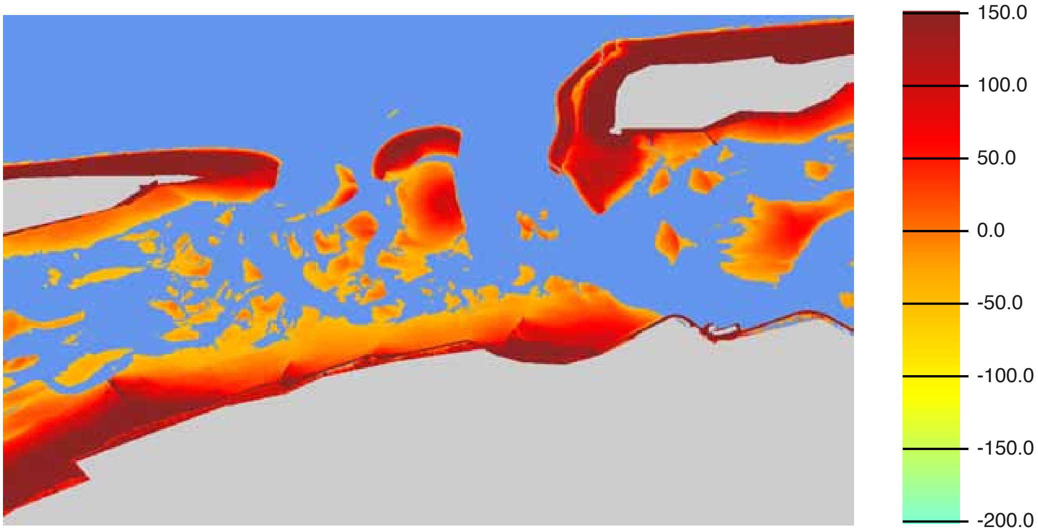

Figure 1.3. Exposure status for the tidal basins "Pinkegat" and "Zoutkamperlaag" calculated for 1-sep-2012, at 2:12. The flooded parts are blue. For exposed parts the selected height map is shown (Deltares Cycle 5 map).

1.2.5 Area statistics

This function returns results for a user-defined grid of points. A grid is defined by a lower-left point and a grid spacing. For all grid points inside the selected area, water levels are calculated and used to estimate a total water volume in the area and a total exposed surface area. These are presented as CSV files and as plots.

The amount of water at each grid point is the amount in excess of NAP (positive) or needed to reach NAP (negative). This means

for points above NAP, the water level used is the height of the water column above the surface.

for points below NAP, the water level used is either the (positive) level above NAP or the negative level below NAP but not further down than the point height.

The (positive or negative) amount of water in each cell is this level multiplied by the cell surface area which is simply the cell distance squared. The total amount for the area is the sum of the amounts calculated for the grid points.

For a maximum of 25400 grid points the calculated water levels and water column heights can optionally be downloaded in "level files". These CSV files can be imported in any spreadsheet program. This allows the subsequent use of biological models for the water column, e.g. light and primary production models.

Some care is required using this option since a dense grid in combination with a period of years easily leads to gigabytes of CSV files.

Figure 1.4 shows an example calculated for subarea Balgzand and based again on there Deltares Cycle 5 height map. The file contains a separate plot for each week in the specified period. The week number on top of each plot complies with the ISO 8601 date and time standard.

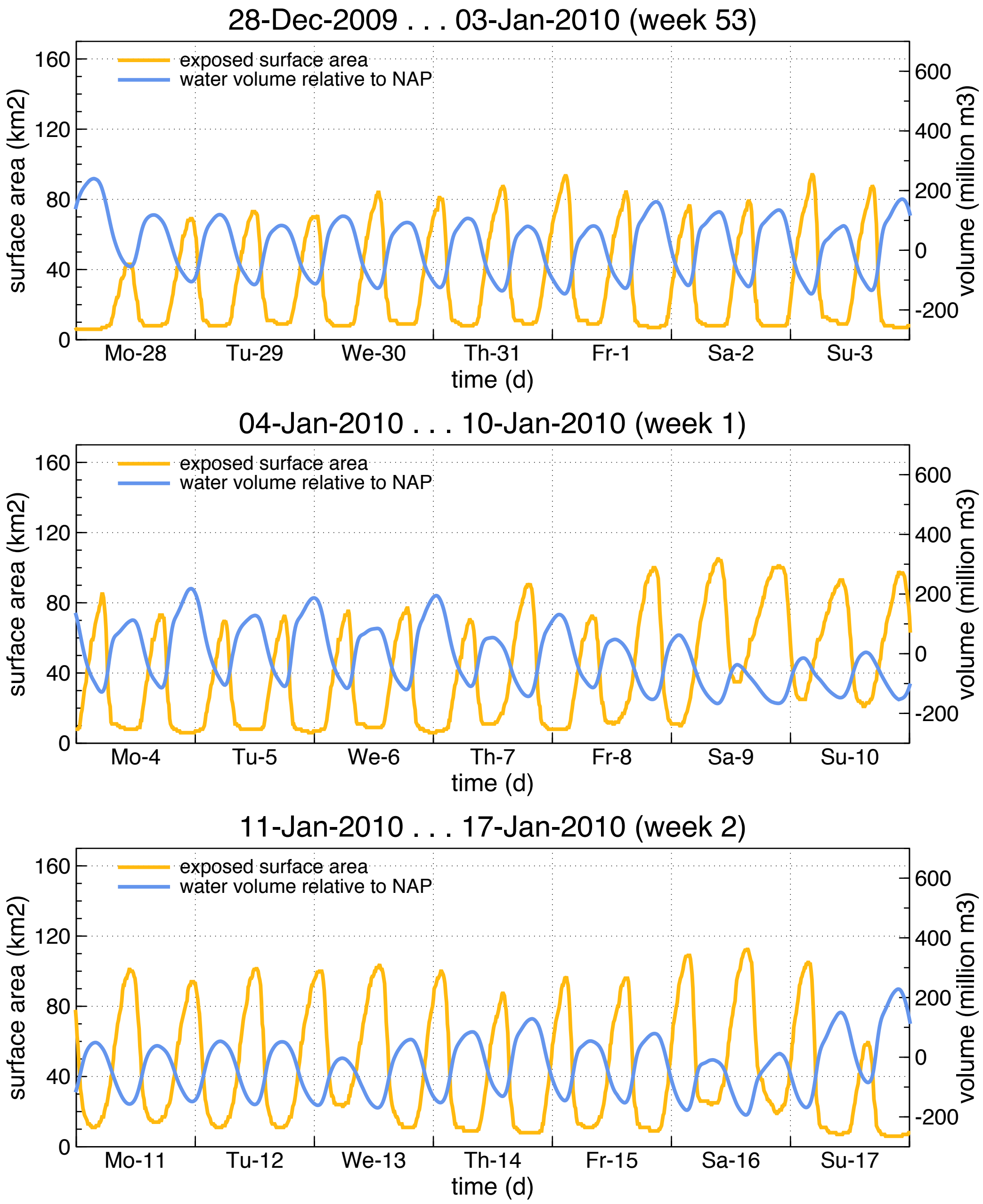

Figure 1.4. Three weeks of exposed surface area and water volume for subarea Balgzand, calculated using the "Area Statistics" function of InterTides. The amount of water is the amount in excess of the NAP level (positive) or required to reach NAP (negative).

1.2.6 Point values

This function requires a number of user-defined points. For each point various quantities are calculated: the distribution of low and high tide water levels and the distribution of times flooded and times exposed (in hours per tidal cycle). These distributions are given as frequency histograms and as CSV files.

The points are specified by the user in a simple CSV file ("Comma Separated Values") which is uploaded automatically. The file must have extension CSV and has three collums named "Name", "RDX" and "RDY". The name will appear in

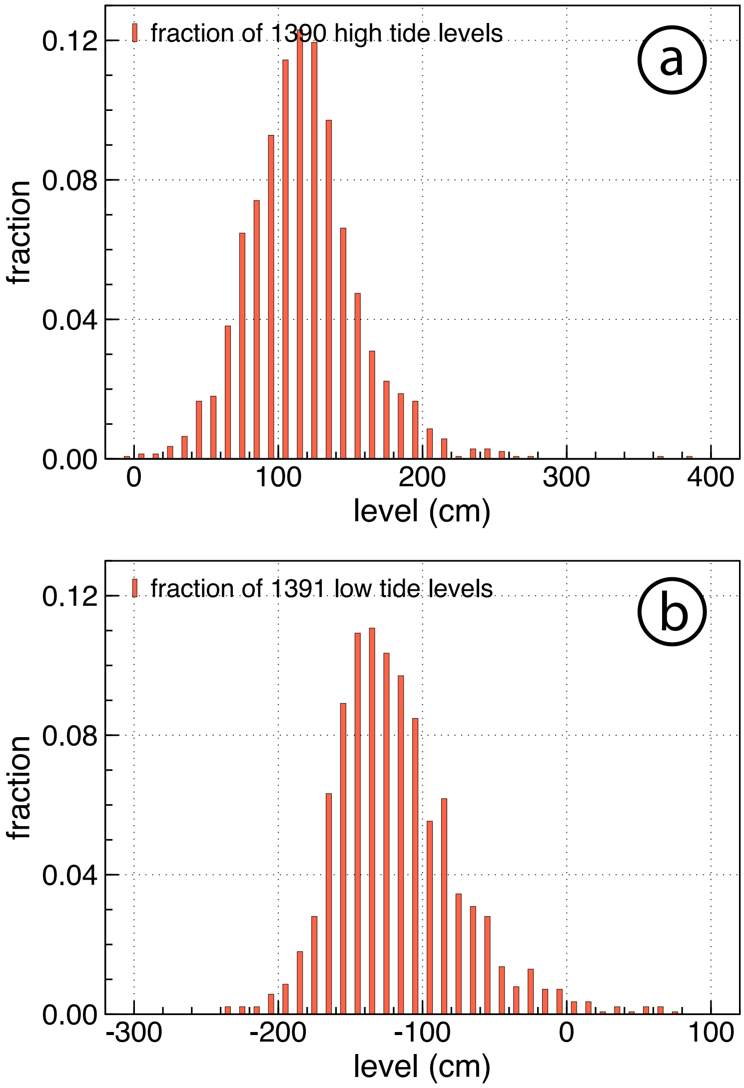

Figure 1.5. The distribution of (a) high tide and (b) low tide levels in november calculated for a point at (228935,607479). The calculations were made for the period [1990, 2013].

the InterTides output and the RDX and RDY columns contain the coordinates. A short example file is

```
Name,              RDX,        RDY
Linthorst,       228935,     605079
Noorderleech ,   177932,     594724
Paesemerlannen,  202879,     602261
Test,            228935,     607479
```

The heights of the points are read from the height map specified in the user interface, just like for the other InterTides functions.

Alternatively, **the uploaded file may have a fourth collumn named "Height"**, which contains the point heights in cm relative to NAP. In this case the height map is neglected.

The calculations are carried out for a chosen period, usually spanning at least one year, and optionally for a season within each year. CSV files with all high tide and

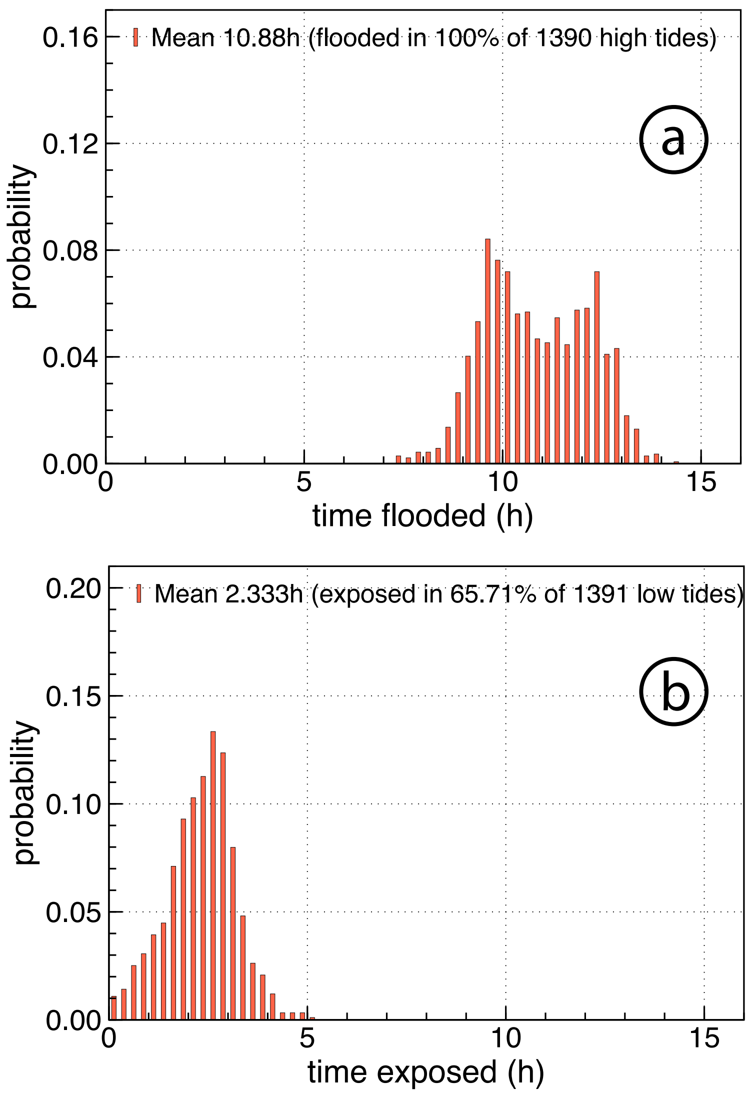

Figure 1.6. The distribution of (a) the time flooded during high tide and (b) the time exposed calculated for the november tides for a point at (228935,607479) during the period [1990, 2013]. The height of the point is -108.4 cm read from the selected map (the Deltares Cycle 5 height map).

low tide times and levels are also constructed, and all calculated water levels can be inspected by optionally writing CSV level files.

As an example we show the results for the point named "Test" in the period [1990, 2013]. An InterTides run was specified with a season between november,1 and november,30. Hence, the results refer to november tides only.

Figure 1.5 shows the distribution of high and low tide levels. Figure 1.6 shows the distribution of flooding and exposure times. Note that a point is not necessarily flooded and exposed during all tidal periods. The legend in Figure 1.6b contains the fraction of low tide periods during which the point is exposed (66%).

There is a pronounced difference between these november plots and results for a month like july (not shown here). This is due differences in astronomical tide and the lack of storm tides.

Chapter 2

Parameters of InterTides

2.1 Height map and related options

Most InterTides functions require the selection of a **HeighMap** from a list of installed height maps. Most height maps will be defined for a cell size of 20 m. If results have to be calculated for a different **Cell size**, the height map is first resized.

This resizing will usually require the combination of the values for several small cells into the value of a single larger cell. The optional parameter **Resize Method** chooses between two methods. The value for the large target cell can be found by calculating a local average value for all small cells inside the target cell. Or by using the nearest neighbour of the target cell midpoint.

In resizing maps it has often to be decided if a large target cell contains a valid value or if it is a NODATA cell. In making that decision InterTides calculates the fraction of the cell covered by data cells from the underlying region map or height map. If this fraction exceeds the value of **Covered Fraction** the cell contains valid data. If not it will be a NODATA cell.

The resizing parameters **Covered Fraction** and **Resize Method** are used for all calculations in which maps with different resolutions have to be combined. Usually a low resolution height map is derived from a high resolution one with 20 m cells. But also in case of a low resolution subarea map projected on the region map, these parameters are used.

2.2 Area selection

The principles of area section were discussed in section 1.1. The are two ways.

- **SubArea** is an optional choice from a predefined list. The list contains parts of the Wadden Sea defined in various ways, as individual muflats, tidal basins, coastal zones or any other way of defining meaningful or practical subareas.
- **Crop** switches on or off the cropping parameters **Crop Lower Left** which is the lower left corner of the cropping rectangle and **Crop XY rectangle**, which specifies the width and height of the cropping rectangle.

The two methods of area selection can be combined. If the cropping rectangle selects part of a subarea, only this part of the subarea is used. If the defined rectangle lies around the subarea, maps calculated for the subarea are augmented with NODATA values. Hence, this is a way in which various subareas can be mapped inside the same user-defined rectangle in subsequent InterTides runs.

2.3 Time, period and season

The first function, water level map, requires just a single **Date** and **Time**. The other functions need a period which is defined using a **Start date** and **Start time** for period begin, and an **End date** and **End time** for period end.

In case of a movie this period will usually span just a few hours or days. There is a maximum number of movie frames which will be reached if you want a movie for a year, for instance. For exposure time maps, area and point functions, the period will usually span several years or even a few decades.

An interesting option of InterTides is that calculations can be limited to a certain **Season**. If this facility is switched on (by setting the switch **Season** on), you have to define a season as two dates, **Season start date** and **Season end date**. Both of them consist of just a day and a month, say 12,05 for May, 12. The season defined in this way is applied to all years in the period. You may calculate for instance an exposure time map for April and May for all years between 1990 and 2010.

A season with its end date *before* its start date defines a winter season. For instance, the season between 01,12 (december 1) and 31,01 (january 31) consists of december and january. Calculations will be limited to these two winter months within the period defined.

2.4 Tide and Interpolation options

If the optional parameter **AdjToNearHT** is switched on, the begin and end time of all calculations is adjusted to the nearest high tide of the region (Harlingen tidal station). This applies both to the a period (spanning years) and to the begin and end of each season within the period. This option guarantees that an integer number of tidal cycles is used in the calculations, which prevents biased results. Therefore the default value of this switch is ON.

The synchronisation option **SyncStations** causes InterTides to synchronize the tidal waves at the various stations. This prevents errors resulting from phase differences between stations used for interpolation. This has been explained extensively in section 3.3. Since tidal waves are seldom symmetrical sine waves, different delays can be used, based on differences between high tide times, low tide times, average differences or the M2 phase. Results calculated for the various delay times will probably differ very little from each other. Without synchronisation, however, considerable errors may be made.

The optional **Sea Level Rise** is just added to all water levels. Hence, it does not change the shape of the tidal curves.

2.5 Output options

The **WaterRange** of **HeighRange** only defines the begin and end points of the colour scale used in images of calculated water level maps or height maps. The ranges do not influence the actual content of the maps.

The option **HmapImage** can be used to switch the generation of a height map image on or off. If many calculations are made for the same height map, you may want to switch this image off.

The option **LevelFiles** causes the generation of large CSV files containing all water levels for the grid points in InterTides function "Area Statistics" or the user-defined points in function "Point Values".

Chapter 3

Interpolation Method

The water level at a certain position and at a certain time is estimated by interpolation between nearby measured levels. By comparing a calculated level with the height read from a height map, the program decides whether or not the point is exposed or not. By repeating the calculations for all measurement intervals of 10 min, we find the fraction of time during which the point is exposed or flooded.

3.1 The data

The blue dots in Figure 3.1 are the tidal gauge stations in the Wadden Sea. From Den Helder along th North Sea coast these are Den Helder, Oudeschild, Vlieland haven, West-Terschelling, Nes, Wierumergronden, Schiermonnikoog en Huibertgat. En from Den Helder along the mainland coast: Den Helder, Den Oever buiten, Kornwerderzand, Harlingen, Lauwersoog, Eemshaven, Delfszijl en Nieuwstatenzijl. At these stations Rijkswaterstaat measures the water level every 10 minutes. Results are freely available from http://live.waterbase.nl.

Figure 3.2 shows an example of measured water levels. The tidal wave moves from west to east and its amplitude increases.

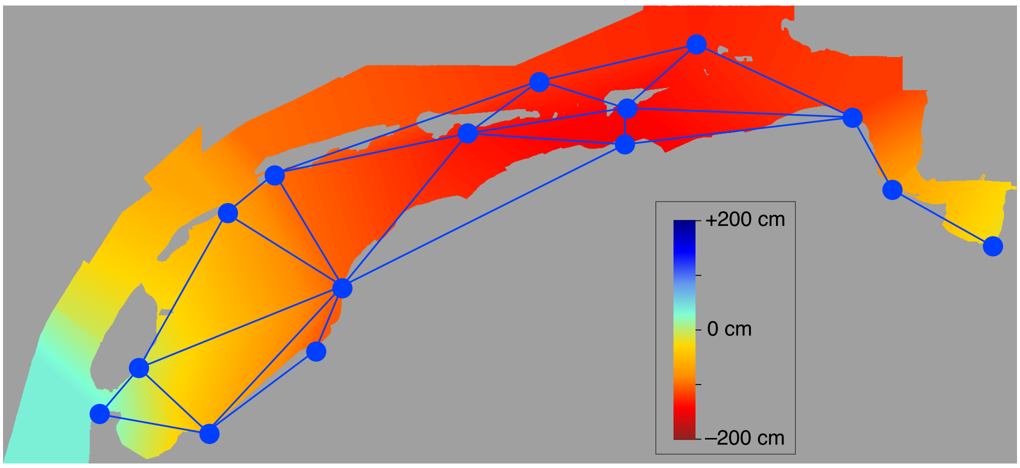

Figure 3.1. Tidal gauge stations of Rijkswaterstaat in de Wadden Sea and an example of interpolated water levels. InterTides interpolates within the triangles covering most of the area. For outside points, the water level is interpolated along the nearest edge.

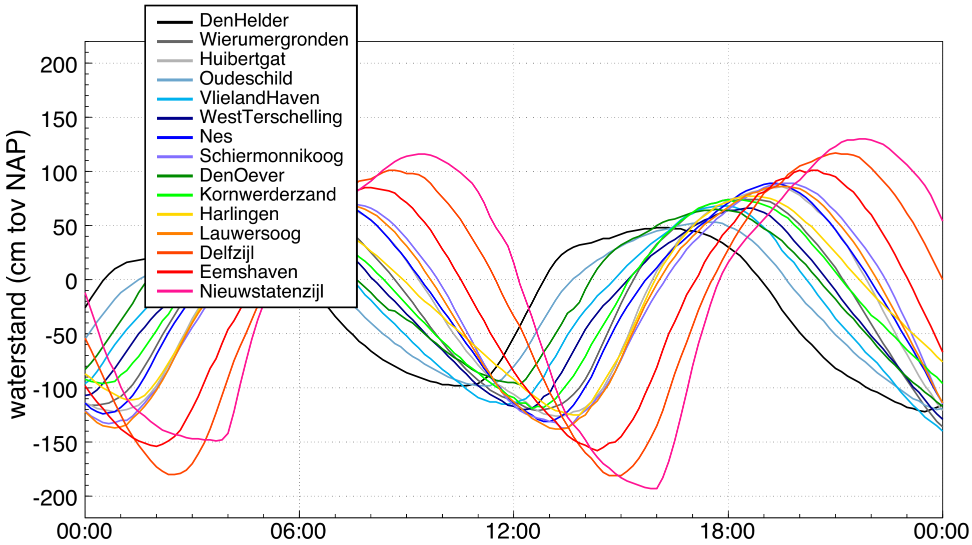

Figure 3.2. Waterlevels of April, 24 2006 measured at the 15 tidal gauge stations of Rijkswaterstaat (data downloaded from http://live.waterbase.nl).

3.2 Spatial Interpolation

The interpolation is carried out in a purely geometrical manner, as a weighted mean of measured water levels. Hence, the hydrodynamics of the various tidal basins is not taken into account. The underlying assumption is that the distances between the gauge stations is sufficiently small.

3.2.1 Triangles

Most of the Wadden Sea is covered by triangles formed by three tidal gauge stations. Inside these triangles water levels are found as a weighted mean of three measured levels.

Figure 3.3a shows a point $\vec{x}$ inside a triangel formed by three tidal stations. The tidal stations have positions given by two dimensional vectors $\vec{S}_1$, $\vec{S}_2$ en $\vec{S}_3$. By shifting th origine to $\vec{S}_1$ we get Figure 3.3b, in which the vectors $\vec{v}_2 = \vec{S}_2 - \vec{S}_1$ en $\vec{v}_3 = \vec{S}_3 - \vec{S}_1$ represent the triangle and point P is given by $\vec{p} = \vec{x} - \vec{S}_1$.

Now, the interpolation formula is found by describing $\vec{p}$ as a linear combination of $\vec{v}_2$ en $\vec{v}_3$. Hence,

$$\vec{p} = w_2\,\vec{v}_2 + w_3\,\vec{v}_3\ . \tag{3.1}$$

For $\vec{x} = \vec{S}_1 + \vec{p}$ this means

$$\vec{x} = w_2\,\vec{S}_2 + w_3\,\vec{S}_3 + (1 - w_2 - w_3)\,\vec{S}_1.$$

Using $w_1 = 1 - w_2 - w_3$ this is equivalent to

$$\vec{x} = w_1\,\vec{S}_1 + w_2\,\vec{S}_2 + w_3\,\vec{S}_3.$$

With these three weights the water level L_P in point P is then found as

$$L_P = w_1\,L_1 + w_2\,L_2 + w_3\,L_3, \tag{3.2}$$

in which L_1, L_2 en L_3 are the three measured levels.

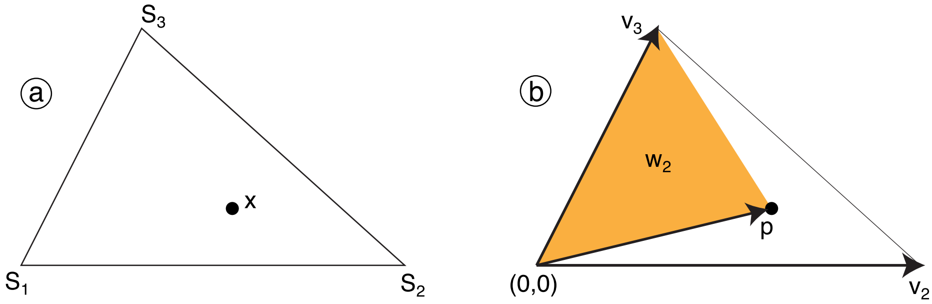

Figure 3.3. A triangel formed by three tidal gauge stations. The water level at a point inside the triangle is calculated as a weighted average of the station levels. The coloured fraction of the triangle surface area is the weight w_2 for station S_2.

The three weights

For finding the three weights Equation (3.1) is written as

$$\begin{cases} p_x = w_2\, v_{2x} + w_3\, v_{3x} \\ p_y = w_3\, v_{2y} + w_3\, v_{3y} \end{cases}$$

with solution

$$\begin{cases} w_2 = \dfrac{(p_x\, v_{3y} - p_y\, v_{3x})}{(v_{2x}\, v_{3y} - v_{2y}\, v_{3x})} \\ \\ w_3 = \dfrac{(v_{2x}\, p_y - v_{2y}\, p_x)}{(v_{2x}\, v_{3y} - v_{2y}\, v_{3x})} \end{cases}$$

De numerator of the expression for w_2 is twice the surface area spanned by the vectors $\vec{p}$ en $\vec{v}_3$. The denominator is twice the surface area of the triangle. Hence, the weight w_2 is equal to the fraction of the triangle surface area covered by the coloured part of Figuur 3.3b. The third weight w_1 is found from $w_1 = 1 - w_2 - w_3$. If all three weights are positive, point P lies inside the triangle.

3.2.2 Interpolation along an edge segment

Around the triangles an edge defined. For a point P (vector $vecx$) not inside one of the triangles in Figure 3.1, the water level is estimated by interpolating between two measured levels, along the nearest line segment of the edge.

The line segments of the edge enclose the area covered by triangles. Their orientation is chosen in such a way that the triangles are kept at the lefthand side of the edge (Figure 3.4). This simplifies the vector calculations.

Moving again the origin to S_1 we define $\vec{v} = \vec{S}_2 - \vec{S}_1$ and $\vec{p} = \vec{x} - \vec{S}_1$. The two weights are then given by

$$\begin{cases} w_2 = \dfrac{p_x\, v_x + p_y\, v_y}{v_x^2 + v_y^2} \\ \\ w_1 = 1 - w_2 \end{cases} \quad . \tag{3.3}$$

If the two weights in Equation (3.3) are positive, point P lies next to the edge segment (S_1, S_2) and the water levels at S_1 and S_2 can be interpolated. There are

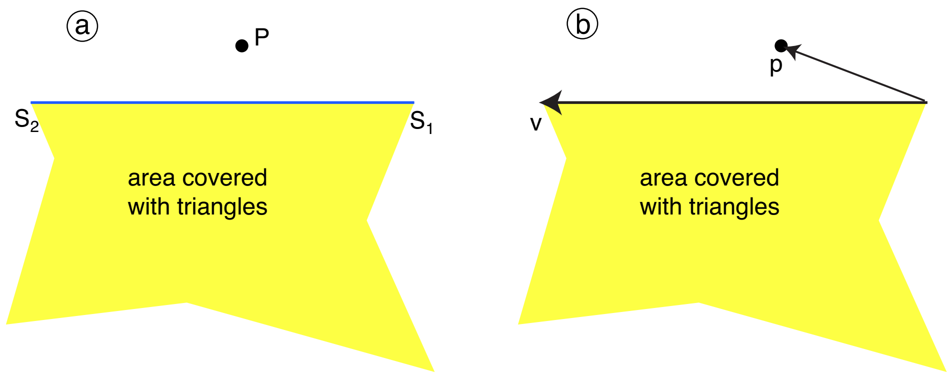

Figure 3.4. An edge segment between two tidal stations S_1 and S_2. If the two weights in Equation (3.3) are positive, point P lies next to this edge segment and the water levels at S_1 and S_2 are interpolated.

additional conditions, however. Obviously P should not be inside any triangle and should lie at the *righthand side* of the selected edge segment. If these conditions cannot be met, there is no suitable edge segment. In that case the nearest tidal station is used, without any interpolation.

3.3 Phase differences

The most simple situation is a tidal wave of uniform amplitude moving through the region. In Figure 3.5 there are two waves with a delay of 2 hours between station 2 and station 1. Now we apply simple linear interpolation to a point in the middle between the two stations and we get the red curve in Figure 3.5. The problem is that the amplitude of this red curve is smaller than the amplitude at the two tidal stations. For a larger delay this damping effect will be larger.

The size of this damping effect can be estimated for the simple situation in Figure 3.5. For a delay of α hours we calculate the amplitude of the interpolated tidal wave $w(t)$. With an equal amplitude A at the stations we get[1]

$$
\begin{aligned}
w(t) &= \frac{A}{2}\left[\sin\left(\frac{2\pi t}{12.421}\right) + \sin\left(\frac{2\pi(t-\alpha)}{12.421}\right)\right] = \\
&= A\cos\left(\frac{2\pi\alpha/2}{12.421}\right) \times \sin\left(\frac{2\pi(t-\alpha/2)}{12.421}\right) = \\
&= A\left[1 - 2\sin^2\left(\frac{\pi\alpha}{24.842}\right)\right] \times \sin\left(\frac{2\pi(t-\alpha/2)}{12.421}\right) = \\
&= A\,[1-\epsilon] \times \sin\left(\frac{2\pi(t-\alpha/2)}{12.421}\right) .
\end{aligned}
\tag{3.4}
$$

The interpolated sine wave $w(t)$ shows a delay $\alpha/2$ relative to station 1, which is fine. Its amplitude, however, is not equal to A, but it is reduced. In Equation (3.4)

[1] This uses $\sin(x) + \sin(x-2y) = 2\cos(y)\sin(x-y)$, in which y is half the phase difference. This can be shown as follows: $\sin(x) + \sin(x-2y) = \sin(x) + \sin(x)\cos(2y) - \cos(x)\sin(2y)$. With $\cos(2y) = 2\cos(y)\cos(y) - 1$ and $\sin(2y) = 2\sin(y)\cos(y)$, the first term cancels and we have $2\sin(x)\cos(y)\cos(y) - 2\cos(x)\sin(y)\cos(y)$, which is equal to $2\cos(y)\sin(x-y)$.

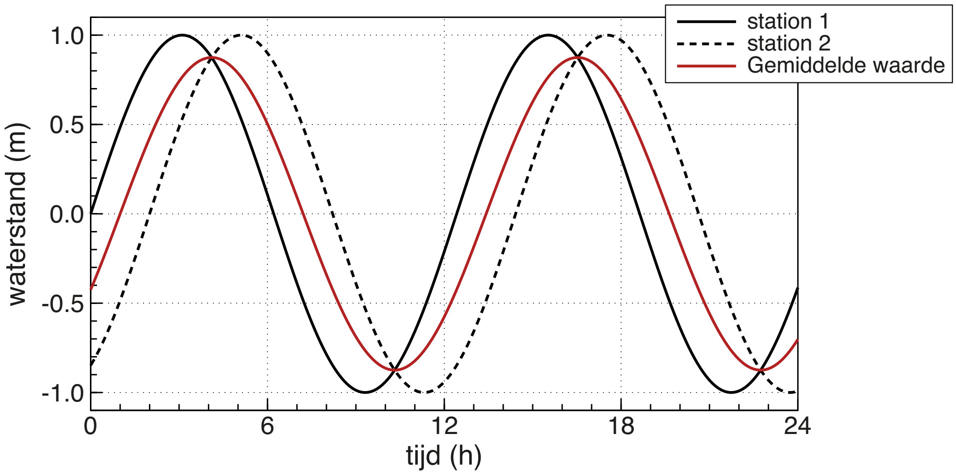

Figure 3.5. The curves for station 1 and station 2 describe a tidal sine wave which needs two hours to move from station 1 to station 2. The red curve is the interpolated water level halfway. The amplitude of the interpolated tide is 13% too small due to the phase difference of 2 hours.

the amplitude of $w(t)$ is written as $A(1-\epsilon)$ and ϵ is then the relative amplitude reduction. From the derivation in Equation (3.4) we can write for ϵ

$$\begin{aligned} \epsilon &= 2\sin^2\left(\frac{\pi\alpha}{24.842}\right) \approx \\ &\approx 2\left(\frac{\pi\alpha}{24.842}\right)^2 \approx \frac{\alpha^2}{31.3}\,. \end{aligned} \tag{3.5}$$

The last two steps are an approximation valid for delays α not exceeding about 3 hours. Equation (3.5) shows that the amplitude reduction increases quadratically with the phase difference between the stations. For $\alpha = 0.5$ hour the deviation is merely 0.8%. For $\alpha = 1.5$ hour the deviation reaches already 7%.

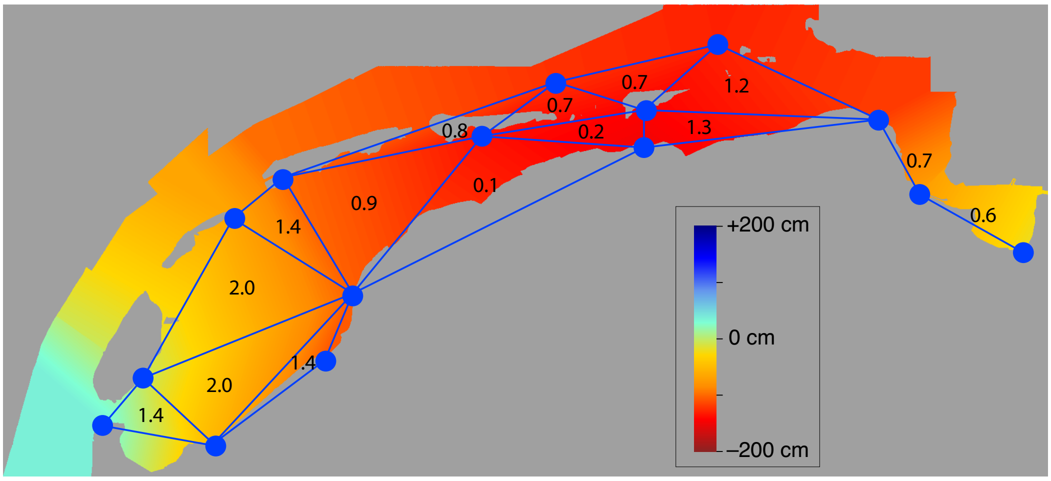

Figure 3.6. Maximum phase difference in hours between two corners of the interpolation triangles. The delays are calculated as the average of the difference in lunitidal interval for high tide and lunitidal interval for low tide (see Table 3.1).

Table 3.1. Lunitidal intervals (or "havengetallen") for the tidal gauge stations. Source: "Waternormalen" or "normaalwaarden" for water levels at the Rijkswaterstaat website www.rws.nl. The tidal waves can be synchronised by using a delay relative to the Harlingen station. The time shift is the delay for high tide, the delay for low tide, or the average of the two, rounded to the nearest 10 min.

Station	Lunitidal interval High tide	Lunitidal interval Low tide	Mean delay (min)
Den Helder	6:11	12:56	−170
Oudeschild	7:12	13:29	−120
Den Oever	7:30	14:20	−90
Vlieland Haven	7:50	14:08	−80
Kornwerderzand	8:17	15:14	−30
West Terschelling	8:21	14:28	−60
Wierumergronden	8:30	14:56	−40
Harlingen	8:37	16:04	0
Lauwersoog	9:18	15:22	0
Huibertgat	9:09	15:34	0
Nes	9:13	15:16	−10
Schiermonnikoog	9:29	15:21	0
Eemshaven	10:20	16:50	+70
Delfzijl	11:05	17:32	+120
Nieuwstatenzijl	11:42	-	+150

The tidal phase differences can be estimated using the lunitidal intervals for the various stations in Table 3.1. Figure 3.6 shows for each of the triangles the maximum delay between any two corners of the triangle. The delays exceed 1 hour for the area west of Harlingen and between Schiermonnikoog and Rottum. The damping errors will be largest there. For the mudflats near Terschelling, Ameland and Engelsmanplaat, these errors will be small.

The damping effect can be largely avoided by synchronising the tidal gauge data before interpolating them. As an example, Figure 3.7a shows again the water levels from Figure 3.2. In Figure 3.7b the curves are shifted using the mean delays in Table 3.1. Figure 3.7c shows the tidal curves synchronized by the phase of the M2 tidal constituent[2]. In this case the lunitidal intervals are not used.

Finally, we need to figure out how to use synchronised levels for the calculation of an interpolated level at a specific point and a specific time. The next section 3.4 shows how this can be done.

3.4 Interpolation between synchronised stations

The basic method remains the same. The water level of a point P at time t is found by interpolating between two or three water levels at nearby tidal stations. The calculations of the weights also remains unchanged.

We write the station weights as w_i, and the water levels as functions of time $W_i(t)$.

[2]M2 is the principal lunar, semi-diurnal constituent. It is calculated by harmonic analysis of the tidal gauge data (see Chapter 4).

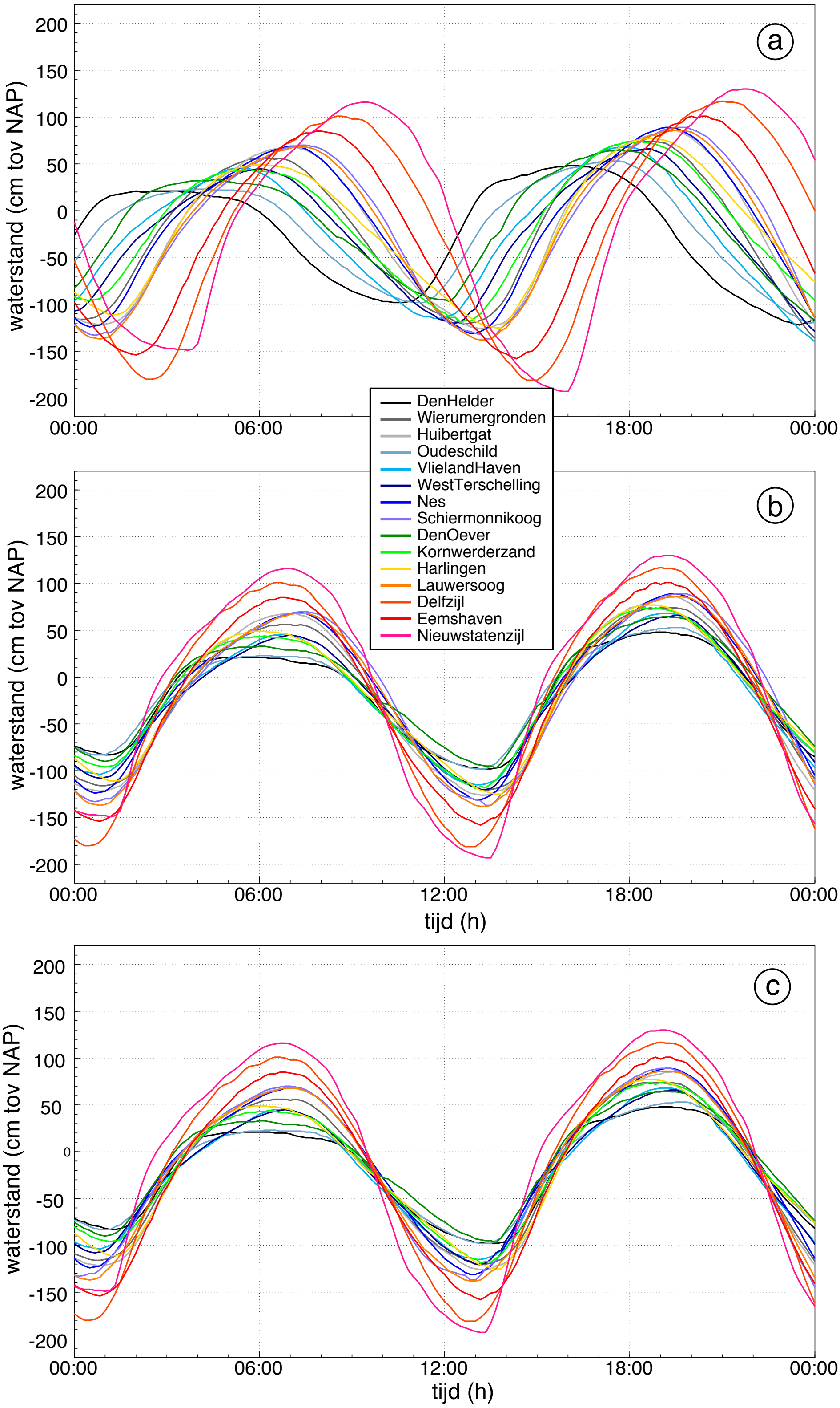

Figure 3.7. Water levels measured at 24 april 2006. (a) As measured by the 15 tidal gauge stations of Rijkswaterstaat, (b) Shifted with the average of the lunitidal interval for high and low tide (see Table 3.1) en (c) Shifted with the phase of the M2 constituent of the station (M2 is the principal lunar, semi-diurnal tidal constituent).

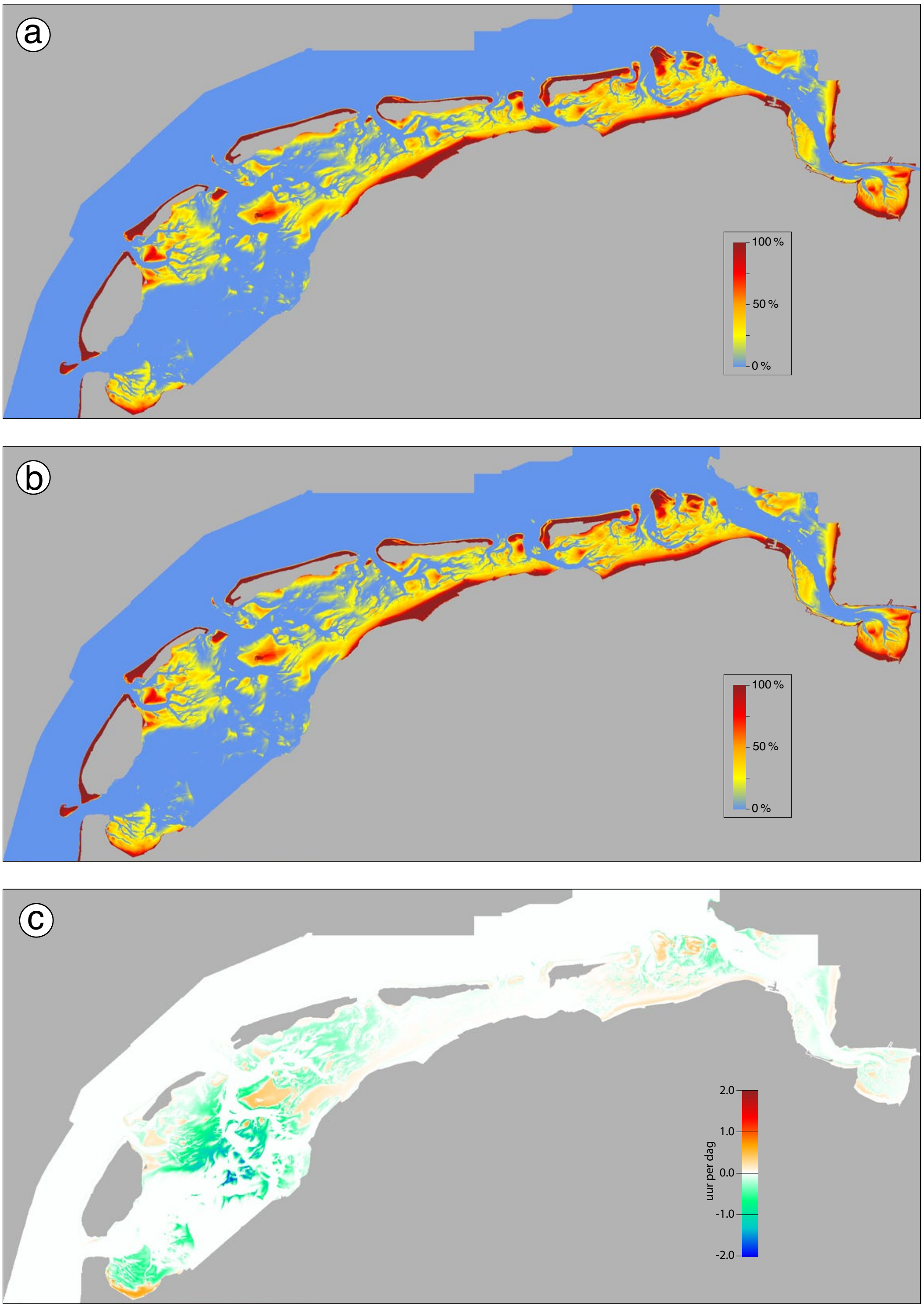

Figure 3.8. Effect of synchronization of tidal gauge stations on a calculated exposure time map. (a) Exposure time as an average percentage of time calculated for non-synchronised tidal data like those in Figuur 3.7a, (b) The same for synchronised data like those in Figuur 3.7b, (c) Difference map with a legend in hours per day. Negative values (green-blue) mean a longer exposure time when synchronization is used.

Without synchronisation we would write for the water level $W_P(t)$ at point P

$$W_P(t) = \sum_i w_i W_i(t). \tag{3.6}$$

With synchronisation we use the delayed levels $W_i(t + d_i)$ in which d_i is the time delay belonging to station i. The water level at point P is then found as

$$W_P = \sum_i w_i W_i(t + d_i), \tag{3.7}$$

but for which time this is the level at point P? A consistent approach is to assume that not just the levels, but also the delays can be interpolated. So the water level W_P in Equation (3.7) actually is the water level at time $t + d_P$ in which d_P is the delay at point P estimated by

$$d_P = \sum_i w_i d_i. \tag{3.8}$$

We now can add the time to W_P in Equation (3.7) and get

$$W_P(t + d_P) = \sum_i w_i W_i(t + d_i), \tag{3.9}$$

which, by subtracting d_P from the time t is equivalent to

$$W_P(t) = \sum_i w_i W_i(t + (d_i - d_P)). \tag{3.10}$$

This Equation (3.10) shows that we first have to calculate d_P by interpolating between the tidal station delays using Equation (3.8). Then we can find the times for the station values as $t + (d_i - d_P)$, hence by correcting t using the *differences in phase* between the point and the stations.

Note that without any phase differences the d_i are zero, therefore also $d_P = 0$ and Equatiion (3.10) reduces to Equation (3.6). Note also that the weighted sum of the phase corrections $\sum_i w_i(d_i - d_P)$ is zero and that the $(d_i - d_P)$ are the *local phase differences*, the differences between the point phase d_P and the phases d_i of the nearby stations. Clearly, in local interpolation only local phase differences should play a role.

3.5 Exposure time map with synchronised stations

The damping effect will affect also exposure times since these are calculated from a large series of interpolated levels. A too small tidal amplitude will cause points with a large height to be flooded less often. This leads to an overestimated exposure time. On the other hand, points with a low height will be exposed less often than by tides with a larger amplitude, which corresponds to an underestimate.

As an illustration of this effect, Figure 3.8a shows a calculated exposure time map without synchronisation, Figure 3.8a with synchronisation and Figure 3.8c is the difference map. The largest deviations are hours per day and indeed occur west of Harlingen and near Rottum, where we would expect them according to Figure 3.6.

InterTides can synchronise the stations using different types of delays, based on high tide times, low tide times, averages of those two, or based on the phase of the M2 constituent calculated for the stations[3]. The delays estimated in different ways will probably lead to very similar results, since a small remaining phase difference will have a negligible effect on interpolated values (e.g. Equation (3.5)).

[3]M2 is the moon tide responsible for most of the tidal wave. This tidal constituent is known as the "principal lunar semi-diurnal" and has an astronomical speed of 28.984104 degree per hour, which corresponds to a period of 12.420601 hours.

Chapter 4

Harmonic analysis

The following background operations of InterTides make use of the harmonic analysis of tidal data.

- **Missing values.** Water levels for the period between 1985 and 1990 are available as hourly values only[1]. The easiest way of dealing with this is to calculate the five missing values per hour as the sum of the "predicted", astronomical level and the interpolated hourly residuals (the difference between measured level and predicted level).
- **First and last day.** Synchronization of the tidal wave at the various stations requires time-shifted water levels. Such a time shift leads to missing data at either the begin or at the end of the entire time series. Harmonic analysis is used to extend the measured time series over an additional day at the begin and at the end of the time range.
- **M2 phase.** One of the synchonization options of InterTides is based on the phase of the M2 constituent (the primary lunar tide). This phase is calculated for each station from the measured water levels in the year 2000.
- **High and Low tides.** The calculation of realised high tide and low tide times and levels by just searching for minimum and maximum water levels is *not* reliable due to storm tides. A better way is to use the tides predicted from harmonic analysis and to search for the real maximum (or minimum) around the astronomical prediction.

In order to accomplish these tasks, InterTides uses a basic form of harmonic analysis as a tool. Since InterTides is not a program dedicated to tidal prediction, the result of the module for harmonic analysis is not available for output. Nevertheless, the InterTides implementation of harmonic analysis will be briefly described.

4.1 Short introduction to harmonic analysis

The periodic change in water level is described as the sum of a series of cosine functions. Each cosine function has an amplitude, a frequency and a phase. The frequencies are known and fixed and the unknown amplitudes and phases need to be determined by harmonic analysis.

[1]The use of a 10-minute interval started around 1990 at different dates for the various tidal gauge stations.

The frequencies depend on a handful of basic astronomical periods (e.g. Kowalik & Luick, 2013; Pugh & Woodworth, 2014) related to earth rotation and the orbits of earth and moon. However, the number of cosine functions (or tidal constituents) required for a satisfactory description of the tide is much larger than the number of basic astronomical periods. There are two reasons for that.

The first reason is that the various astronomical effects do not lead to additive and independent tidal waves. The amplitude of principal lunar tide, for instance, changes periodically due to the fact that the orbit of the moon is an ellipse and not a circle. Mathematically a cosine function with a slowly but periodically changing amplitude is equivalent to the sum of *three cosine functions* with constant amplitudes and slightly different frequencies. Such interactions between the various astronomical periods lead to tens of larger and smaller tidal constituents.

The second reason is that the shape of the tidal wave changes in shallow water like the North Sea coast and the Wadden Sea. If a cosine function changes in shape it obviously is no longer a cosine function, but its frequency f is preserved. Mathematically this means it can be described as the sum of cosine functions with frequencies f, $2f$, $3f$, $4f$, etc. Analogous to the overtones in music, the additional higher frequency constituents are known as "shallow water overtides". The higher order constituents become smaller and smaller.

Harmonic analysis means that the amplitudes and phases of the important constituents are determined from a series of measured water levels. Once we have these amplitudes and phases a water level can be calculated for a time in past or future by just calculating all constituents for that time and summing them. Hence, the prediction of tides is little more than solving a fitting problem and an easy way is by linear regression analysis.

The important tidal constituents have been named and classified by Doodson (1921). His system has been extended, but is still in use today (e.g. `http://www.iho.int/mtg_docs/com_wg/IHOTC/IHOTC_Misc/TWLWG_Constituent_list.pdf`).

4.2 Harmonic analysis by InterTides

The constituents used are the same as in the predefined list for a year of data in `http://apps.helpdeskwater.nl/downloads/extra/simona/release/doc/usedoc/tidegui/a872r1r11.pdf`. This list contains the average water level and 94 tidal constituents.

Each constituent (cosine function) with unknown amplitude and phase can be rewritten as the sum of a sine and a cosine function, both with an amplitude only (e.g. Kowalik & Luick, 2013). This allows us to write the harmonic analysis as a linear regression problem with $1 + 2 \times 94 = 189$ unknown parameters. In order to solve this problem InterTides calls Lapack[2] subroutine "gelss" from the Math Kernel Library of Intel (`https://software.intel.com/sites/products/documentation/hpc/mkl/mklman/`). This subroutine calculates the least squares solution after singular value decomposition of the coefficient matrix.

All calculations are carried out with the time in days starting at 31-dec-1899 00:00:00[3]. Table 4.1 at page 29 gives an example of the results. It shows the harmonic constituents calculated for station Den Helder for the year 2000.

[2] Linear Algebra Package

[3] This zero point can also be set in Excel spreadsheets, but we skip 29-feb-1900 as a day and Excel does not (in Excel the year 1900 is incorrectly treated as a leap year).

Table 4.1. Harmonic analysis for Den Helder for the year 2000 using the 94 Rijkswaterstaat constituents. The fitted mean sea level was 2.8 cm +NAP and the residual error of the regression analysis is 24 cm. De phases are in degrees.

Constituent Name		Speed (deg/hr)	Amplitude (cm)	Phase (deg)
1	Sa	0.0411	10.8	338.0
2	SM	1.0159	2.2	65.0
3	Q1	13.3987	2.9	49.1
4	O1	13.9430	9.7	169.0
5	M1C	14.4921	1.4	323.0
6	P1	14.9589	3.4	12.9
7	S1	15.0000	1.2	144.4
8	K1	15.0411	6.9	6.7
9	3MKS2	26.8702	1.1	146.5
10	3MS2	26.9523	1.9	306.9
11	OQ2	27.3417	0.4	294.9
12	MNS2	27.4238	1.4	130.9
13	2ML2S2	27.4967	1.0	232.9
14	NLK2	27.8861	3.1	62.7
15	mu2	27.9682	8.0	219.7
16	N2	28.4397	10.1	74.1
17	nu2	28.5126	3.8	155.4
18	MSK2	28.9020	1.0	142.3
19	MPS2	28.9430	2.4	54.6
20	M2	28.9841	65.9	160.4
21	MSP2	29.0252	0.6	264.1
22	MKS2	29.0662	0.2	145.1
23	lambda2	29.4556	3.1	301.3
24	2MN2	29.5285	6.3	58.9
25	T2	29.9589	1.0	215.0
26	S2	30.0000	17.9	256.8
27	K2	30.0821	4.8	76.1
28	MSN2	30.5444	1.8	138.6
29	2SM2	31.0159	2.2	122.6
30	SKM2	31.0980	1.3	311.8
31	NO3	42.3828	0.3	169.7
32	2MK3	42.9271	0.9	275.9
33	2MP3	43.0093	0.2	275.2
34	SO3	43.9430	0.3	331.1
35	MK3	44.0252	0.4	196.7
36	SK3	45.0411	0.1	319.3
37	4MS4	55.9364	0.5	310.2
38	2MNS4	56.4079	0.5	126.8
39	3MS4	56.9523	1.9	219.6
40	MN4	57.4238	3.9	71.0
41	2MLS4	57.4967	1.4	151.9
42	2MSK4	57.8861	1.0	189.5
43	M4	57.9682	11.7	158.7
44	3MN4	58.5126	1.9	56.1
45	MS4	58.9841	6.4	248.5
46	MK4	59.0662	2.1	73.1

continued on next page

Table 4.1. *continued*

	Constituent Name	Speed (deg/hr)	Amplitude (cm)	Phase (deg)
47	2MSN4	59.5285	1.2	145.8
48	S4	60.0000	0.5	3.6
49	MNO5	71.3669	0.6	95.3
50	3MK5	71.9112	1.1	205.6
51	2MP5	72.9271	0.6	325.6
52	3MO5	73.0093	1.1	28.1
53	MSK5	74.0252	0.5	107.8
54	3KM5	74.1073	0.1	284.4
55	3MNS6	85.3920	0.7	234.9
56	2NM6	85.8636	1.1	67.8
57	4MS6	85.9364	1.1	313.5
58	2MN6	86.4079	3.3	161.0
59	2Mnu6	86.4808	1.1	256.8
60	3MSK6	86.8702	0.7	250.5
61	M6	86.9523	5.8	256.5
62	MSN6	87.4238	1.1	267.2
63	MKnu6	87.5788	0.2	197.7
64	2MS6	87.9682	5.5	340.3
65	2MK6	88.0503	1.5	149.5
66	3MSN6	88.5126	1.2	246.6
67	2SM6	88.9841	1.0	73.6
68	MSK6	89.0662	0.5	256.7
69	2MNO7	100.3510	0.2	161.7
70	M7	101.4490	0.2	333.1
71	2MSO7	101.9112	0.3	16.5
72	2(MN)8	114.8477	0.8	82.1
73	3MN8	115.3920	1.8	190.8
74	M8	115.9364	2.2	289.3
75	2MSN8	116.4079	1.1	290.6
76	2MNK8	116.4901	0.9	89.0
77	3MS8	116.9523	3.3	11.9
78	3MK8	117.0345	0.9	190.2
79	2(MS)8	117.9682	1.2	109.8
80	2MSK8	118.0503	0.5	300.6
81	3MNK9	130.4331	0.2	56.4
82	4MK9	130.9775	0.1	161.9
83	3MSK9	131.9934	0.2	221.8
84	4MN10	144.3761	0.7	267.5
85	M10	144.9205	0.6	16.4
86	3MSN10	145.3920	0.8	357.9
87	4MS10	145.9364	1.3	86.7
88	2(MS)N10	146.4079	0.2	146.3
89	3M2S10	146.9523	0.7	171.1
90	4MSK11	160.9775	0.1	308.9
91	M12	173.9046	0.1	97.6
92	4MSN12	174.3761	0.4	44.6
93	5MS12	174.9205	0.4	132.3
94	4M2S12	175.9364	0.4	220.4

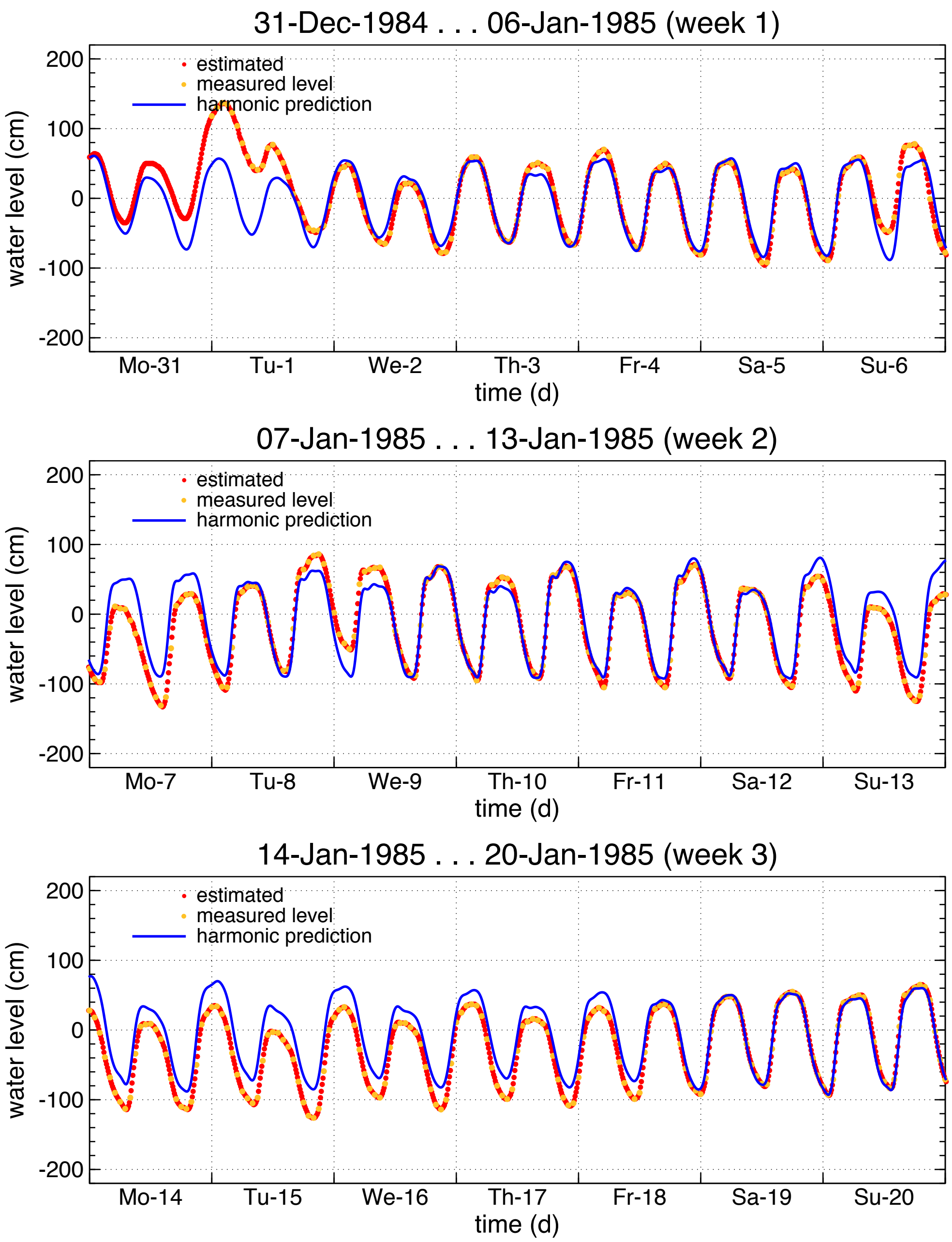

Figure 4.1. The tidal gauge time series for Den Helder begins on Tuesday January,1 1985 at 00:00. The available measured levels are hourly values and the five missing values per hour were estimated by means the calculated astronomical tide (the blue curve "harmonic prediction"). Also the levels for December,31 were estimated from the harmonic prediction.

No attempt is made to identify constituents which are not statistically significant. We just use the 94 constituents to calculate astronomical water levels at arbitrary times.

Figure 4.1 shows an example of the application. The blue curve is the astronomical tide and the yellow points are measured water levels. Clearly, the measured levels deviate from the "prediction" which is mainly due to the influence of weather and atmospheric pressure. The hourly residuals can be interpolated however and added to the astronomical tide. This leads to realistic intermediate values.

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