THE UPWARD-DRIVEN PENDULUM

A (mechanical) toy model of convection

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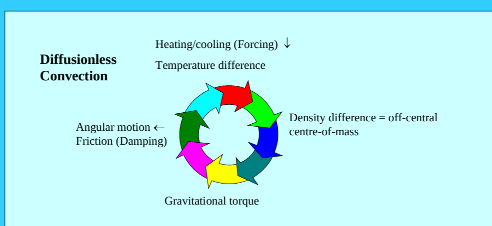
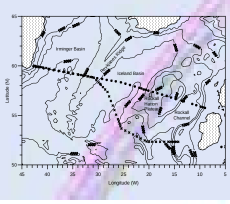
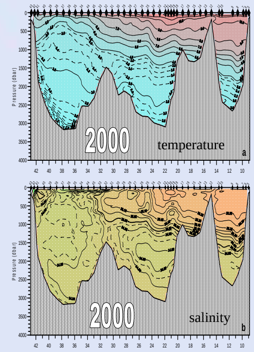
To the mathematical notion of 'toy model', such as the DLE for diffusionless convection below, we here add its mechanical equivalent, literally made of LEGO-toys: the upward-driven pendulum.

This pendulum aims to show the essence of convection as captured by the dynamics of a fluid's centre-of-mass and reveals both periodic and aperiodic (chaotic) behaviour.
It explains the importance of preconditioning (near absence of lateral density gradients) for generation of the equivalent of a 'deep flush'.

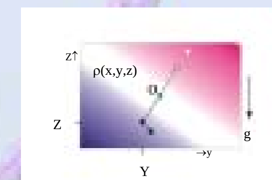
Heating of fluid from below ('pan on fire'), or cooling from above (e.g. Labrador Sea) → heat transport by

- radiation
- molecular diffusion
- fluid motion = convection



Measurements: H.M. van Aken



Centre-of-mass (X,Y,Z)=∫(x,y,z) ρ dV

Sea is density stratified ρ; warm/fresh=light; cold/salt=heavy

Model dynamics of stratified sea by centre-of-mass ***X**(t)* = ∫***x*** ρ dV
Coupled to basin-averaged angular momentum ***L** (t)* = ∫***x*** ×***u*** dV
In rotating 3D rectangular box (Maas 1994):

$$\frac{d\boldsymbol{X}}{dt} + \boldsymbol{X} \times \boldsymbol{L} = -\boldsymbol{X} + \mathrm{Ra}\ \boldsymbol{F}$$

$$\mathrm{Pr}^{-1}\frac{d\boldsymbol{L}}{dt} + f\mathbf{k} \times \boldsymbol{L} = -Y\,\mathbf{i} + X\,\mathbf{j} - \boldsymbol{L} + \boldsymbol{T}$$

Pr, Ra: Prandtl, Rayleigh number
F,***T***: differential buoyancy & momentum flux through boundaries
f: Coriolis parameter
i,j,k: unit vectors in x,y,z direction

In absence of rotation (f=0) → transformed version of Lorenz-63 equations

$X \equiv L_1 \quad Z \to r - Z, \mathrm{Ra} = r\mu$

- explicit forcing
- simple phase space
- gravity points down (Lor63: 'upside down'...)
- 2 damping mechanisms: friction & diffusion

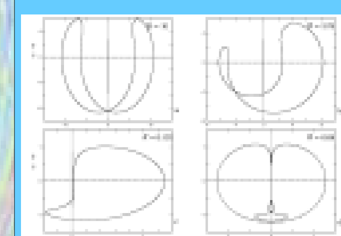
$$\mathrm{Pr}^{-1}\frac{dX}{dt} + Y = -X$$
$$\frac{dY}{dt} + XZ = -Y$$
$$\frac{dZ}{dt} - XY = -\mu Z + \mathrm{Ra}$$

No diffusion → 1 parameter Diffusionless Lorenz Equations (DLE)

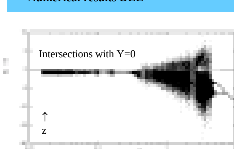
$\mathrm{Pr}, \mathrm{Ra} \to \infty, \mathrm{R} = \mathrm{Ra}/\mathrm{Pr}^2$ finite
$(Y, Z) \to (-Y, Z)/\mathrm{Pr}$

Van der Schrier and Maas 2000

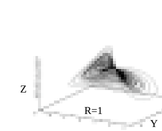
$$\frac{dX}{dt} = Y - X$$
$$\frac{dY}{dt} = XZ$$
$$\frac{dZ}{dt} = -XY + \mathrm{R}$$

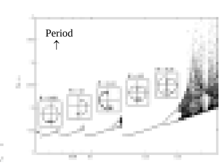


Numerical results DLE



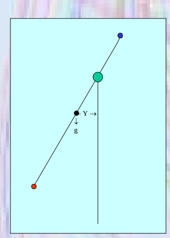
-dominance of Shilnikov bifurcations from R→∞ down:
-periodic orbit,
symmetry breaking,
-period doublings,
chaos, reverse bif.,
-self-similar copies

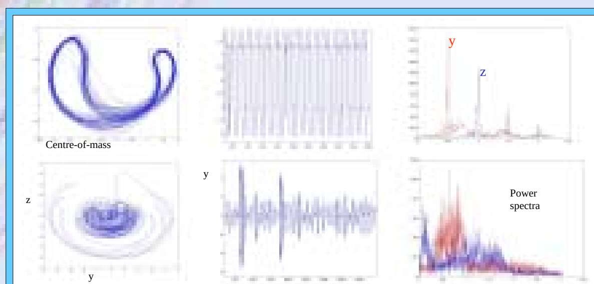




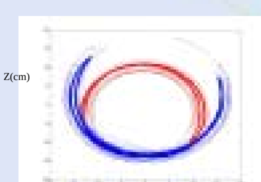
Mechanical convection described by upward-driven pendulum

Gears drive pendulum upward (strength adjustable). Difference with fluid: centre-of-mass goes upward *through* instead of *along* centre. If centre-of-mass ascends above centre, gravitational torque topples pendulum; the smaller Y, the longer it takes. When centre-of-mass descends below centre, gear direction is changed.





2 regimes in pendulum dynamics for 2 different speeds of ascend



Example of motion of red and blue endpoints in Periodic regime

References
Maas LRM 1994 Tellus 46A, 671
van der Schrier, G and LRM Maas 2000 Physica D 141, 19

Possible future work
- construction bifurcation diagram of upward-driven pendulum
- adding rotation