Experimental verification of Lorentz' linearization procedure for quadratic friction

Guido M. Terra,[a] Willem Jan van de Berg,[b] and Leo R.M. Maas
Royal Netherlands Institute for Sea Research, Physics Department,
P.O. Box 59, NL-1790 AB Den Burg, The Netherlands.
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In this article we present results from laboratory experiments with a "tidal tank". It demonstrates the resonant response of the Helmholtz mode in an almost enclosed basin. In particular the dependence of the response curve (amplification of the tide in the basin relative to the tidal signal at sea) on the tidal amplitude due to the nonlinear influence of dissipation is observed. Although Lorentz' linearization procedure for coping with this nonlinear effect was already developed at the beginning of the previous century, quantitative experimental validation in this context has not yet been provided.

I. INTRODUCTION

Tidal motion is a key feature in many coastal areas. Both the vertical motion, changing the water depth, and the horizontal currents have to be taken into account for navigational purposes. Tidal currents transport loads of sediment, changing the coastal bathymetry, and play an important role in flushing the basins, distributing biological and chemical agents. They are caused by the attraction of the sun and the moon. However, direct tidal forcing accounts for no more than a lunar M_2-tide (period 12.42h) of 27 cm and a solar S_2-tide (period 12h) of 13 cm amplitude (the *equilibrium tide* in case the earth were completely covered by water, neglecting inertia). Extreme tidal amplitudes are found e.g. in the Bay of Fundy (up to 8 m)[1], in the Sea of Ochotsk and near the coast of Normandy (over 6 m). Typically, tidal ranges of a couple of meters are found in the North Sea and the Dutch Wadden Sea, for example.

The amplification of the equilibrium tide in coastal areas is a result of resonance. The directly driven ocean tide propagates from the oceans into cooscillating coastal seas. Amplification occurs in the process if the tidal period is close to one of the eigenmodes of the basin, such as the *quarter wavelength* modes in half open channel-like basins or the *Helmholtz* or *pumping mode* in almost-enclosed basins. Linear theory describing this is well established and can be found in several textbooks[2–4]. Observations are explained fairly well. For example, Garrett[5] shows that the eigenperiod of the Bay of Fundy together with the adjacent Gulf of Maine is 13.3h, in near resonance with the dominant M_2-tide from the Atlantic Ocean.

The Helmholtz mode occurs in relatively small tidal basins cooscillating with the sea/ocean through a narrow inlet. It is characterized by uniform sealevel elevation within the basin. The balance between the inertia of the flow through the inlet and the restoring force due to the sealevel difference between that at sea and in the basin, determines the oscillation[6–8]. In their models, only (linear) damping due to radiation of wave energy into the surrounding ocean is incorporated. This leads to the so-called harbor paradox[9,10]. Intuitively one would expect suppression of the response to external forcing when decreasing the width of the inlet, due to increased screening. However, the maximum and mean square response appear to increase. This is explained by the fact that radiation of energy is reduced. Linear theory has also been tested in the laboratory. McNown[11] studied the structure of the eigenmodes in circular basins, similar experiments have been done for rectangular basins[12,13]. Lee[14] measured the response curves of almost enclosed circular basins, a half open channel and a more complex model of the Long Beach Harbour. Harbours with multiple basins were studied in more detail by Lee and Raichlen[15]. All of them found the linear response curves for these basins without considering the influence of the tidal amplitude.

Nonlinear effects arise due to the advective terms in the momentum equation, sealevel variations in the continuity equation, non-uniform hypsometry (tidal flats cause the basin's wet area to change with waterlevel) and quadratic friction (either by bottom stresses or by head loss effects). In observational studies nonlinear effects manifest themselves in the generation of higher harmonics[16–18], drifting harmonic "constants"[19,20] and chaotic behaviour[21,22].

Miles[23] extended his linear theory for the Helmholtz mode[6] to include the effect of changing channel cross section with sealevel. This shortens the high water period and increases the low water period. Moreover, the total period of free oscillations increases with amplitude and the response curve is tilted. As a result multiple equilibria are found: for the same tidal signal at sea, different tidal response amplitudes in the basin are possible. Elaborating upon the work of Green[24], it was shown[8] that non-uniform hypsometry (i.e. the dependence of basin area on waterlevel) has a similar

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[a] Electronic mail: `terra@nioz.nl/gmterra@science.uva.nl`; University of Amsterdam, Korteweg-de Vries Institute for Mathematics, Plantage Muidergracht 24, NL-1018 TV Amsterdam, The Netherlands.

[b] Institute for Marine and Atmospheric research Utrecht, P.O. Box 80000, NL-3508 TA Utrecht, The Netherlands.

but opposite effect: reduction of the low water period and lengthening of the high water period. In the limit of small friction and near resonance, he found multiple equilibria as well. Moreover, if the tidal signal at sea is not purely sinusoidal but consists of a couple of nearly resonant components (such as M_2 and S_2, causing the spring-neap tidal cycle) chaotic behaviour of the response amplitude was shown to occur[25,26]. Although this theory was developed for the Helmholtz mode in an almost enclosed basin, it has been extended to higher modes in more general basins as well by Terra et al.[27].

Dissipation of energy in resonant harbours is through radiation damping, bottom friction and *head loss* (pressure drop) at the entrance due to flow separation. Radiation damping is adequately described linearly[4,6,28,29] and is corrected for in our experiments (see the remark at the end of section III). Bottom friction is commonly modelled by the Chezy law, which actually has the same form as the head loss term: using hydraulic theory it can be shown that the head loss term depends quadratically on the fluid velocity[8,30–32]. In order to evaluate the influence of the quadratic (bottom) friction term Lorentz[33] proposed to replace it by a linear term with the friction coefficient chosen such that the energy dissipation per tidal cycle would be the same as if the nonlinear law were used. Zimmerman[34] interpreted it as a renormalization procedure. Mei[35] mentions Lorentz' linearization method under the name of *equivalent linearization*. Zimmerman[29] applied the method to the special case of the Helmholtz mode, in which case the results are particularly simple. A more elaborate exploration without assuming the tide to be uniform inside the basin is performed by Ünlüata and Mei[36] and Gerber[37], thus including the sloshing modes as well. They conclude that the Helmholtz mode is affected most by the head loss effects. Dronkers[38,39] and Le Provost[40] give a complete expansion of the quadratic friction term, which is used in a perturbation method by Kabbaj and Le Provost[41]. All these methods are virtually equivalent, at least to first order. Hence the simplest approach of Zimmerman[29] is followed in this paper.

Incorporation of *head loss* solves the harbour paradox. Indeed, based on computer simulations Ito[42] reports that the vulnerability of the Port of Ofunato to tsunamis decreased considerably due to the partial closure of the entrance by a breakwater and found the head loss coefficient for which his numerical simulations agree reasonably with observations from tide gauges. However, experiments to corroborate Lorentz' linearization theory seem to be missing. Although Horikawa and Nishimura[43] do present response curves for different forcing amplitudes, they rate them as less accurate owing to the deformation of the waves and do not provide an adequate quantitative comparison with theory. Both Bowers[44] and De Girolamo[45] state qualitatively that the head loss effect is found to cause the amplification factor to decrease with increasing amplitude in their preliminary experiments. However, their main focus is on the generation of harbour oscillations due to the nonlinear setup/setdown by wave groups. Although they do mention the quadratic dependence of the energy loss on velocity, no quantitative analysis is given by them either. In particular, no reference to Lorentz' linearization principle is made.

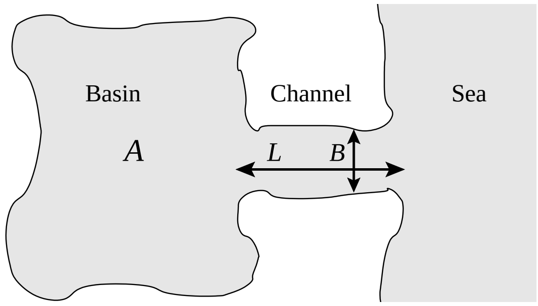


FIG. 1: Top view sketch of an almost enclosed tidal basin with horizontal area A, cooscillating with the tide at sea through an inlet channel of width B, length L and depth H.

In order to fill the lack of experimental validation of Lorentz' linearization principle, we present a quantitative comparison between our measurements and the results of a simple model for the Helmholtz mode with Lorentz' linearized friction. In this paper a brief overview of the application of Lorentz' linearization theory to the Helmholtz mode in an almost enclosed basin is given in section II. Subsequently, the experimental setup is described in section III. Experimental results are discussed in section IV and compared with the theoretical response curves derived in section II. Finally, the paper is concluded with a short summary and discussion of forthcoming work.

II. THEORY

Consider an almost enclosed basin with horizontal area A connected to the sea by a channel of length L, width B and depth H, as shown in Fig. 1, which is forced by a tidal signal at sea with angular frequency ω. Its dimensions are assumed to be much smaller than the tidal wavelength $\lambda = 2\pi\sqrt{gH}/\omega$, such that the tidal wave traverses the basin "instantaneously". Hence the waterlevel in the basin can be described by a single value ζ, the elevation relative to the still water level. Water flows into the basin through the channel with velocity u. All dynamics is concentrated in the channel, where the elevation difference between sea and basin drives the flow, balanced by the inertia of the water in the channel and bottom friction/head loss. Bottom friction is commonly described by Chezy's law $(c_D/H)\,|u|u$, with drag coefficient $c_D \approx 0.0025$ determined empirically[46]. The head loss due to flow separation is parameterized by $(f/L)\,|u|u$ and differs only in the formulation of the coefficient; f has to be determined empirically as well. Ito[42] found that $f \approx 1.5$ gave the best results in his numerical simulations of the Port of Ofunato by comparing them with observations from tide gauges. The dynamics of the system can be described by the equations for momentum and mass conservation:

$$\frac{\mathrm{d}u}{\mathrm{d}t} = \frac{g}{L}\,(\zeta_e - \zeta) - \frac{f}{L}\,|u|\,u, \tag{1a}$$

$$A\,\frac{\mathrm{d}\zeta}{\mathrm{d}t} = O\,u, \tag{1b}$$

where $\zeta_e = \alpha_e \cos(\omega t) = \mathrm{Re}[\alpha_e\,\mathrm{e}^{\mathrm{i}\omega t}]$ is the prescribed elevation at sea and $O = BH$ is the channel cross section. The head loss coefficient f/L is considered to include bottom friction as well (f/L "=" $f/L + c_D/H$). Note that radiation damping is not modelled here explicitly but is assumed to be already incorporated in ζ_e. In analyzing the experimental results we will do the same.

The Lorentz' linearization principle amounts to replacing the nonlinear head loss term $(f/L)\,|u|u$ by ru, with effective linear friction coefficient r to be determined. In that case the system (1) is linear and the response will be of the form $\zeta = |\alpha|\cos(\omega t + \varphi) = \mathrm{Re}[\alpha\,\mathrm{e}^{\mathrm{i}\omega t}]$. and consequently $u = -(A/O)\,\omega|\alpha|\sin(\omega t + \varphi)$. Lorentz' energy principle states that the energy dissipation per tidal cycle should be the same for both formulae: $\langle ru^2\rangle = \langle (f/L)\,|u|u^2\rangle$, where $\langle\cdot\rangle$ denotes averaging over a tidal cycle. Evaluation of the corresponding integrals leads to $r = \nu_0\,\omega|\alpha|$, with $\nu_0 = \frac{8}{3\pi}\frac{fA}{OL}$, i.e. effective friction increases linearly with tidal amplitude in the basin. Using the linear expression for friction, (1) can be simplified to

$$\frac{\mathrm{d}^2\zeta}{\mathrm{d}t^2} = {\omega_0}^2\,(\zeta_e - \zeta) - \nu_0\,\omega|\alpha|\,\frac{\mathrm{d}\zeta}{\mathrm{d}t},$$

in which ${\omega_0}^2 = gO/(AL)$ is the eigenfrequency of the Helmholtz mode. Substituting $\zeta = \mathrm{Re}[\alpha\,\mathrm{e}^{\mathrm{i}\omega t}]$ the complex response equation is found:

$$({\omega_0}^2 - \omega^2)\,\alpha + \mathrm{i}\,\nu_0\,\omega^2\,|\alpha|\alpha = {\omega_0}^2\,\alpha_e. \tag{2}$$

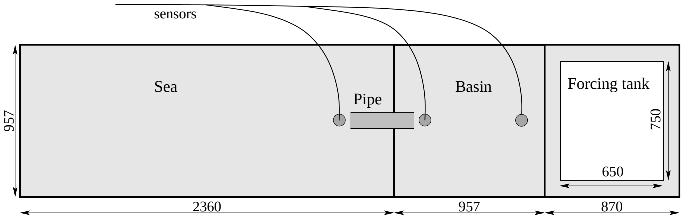
By taking the square modulus of this equation it is possible to solve for $|\alpha|$ and find

$$\frac{|\alpha|}{|\alpha_e|} = \sqrt{\frac{\sqrt{(1-(\frac{\omega}{\omega_0})^2)^4 + 4{\nu_0}^2(\frac{\omega}{\omega_0})^4|\alpha_e|^2} - (1-(\frac{\omega}{\omega_0})^2)^2}{2{\nu_0}^2(\frac{\omega}{\omega_0})^4|\alpha_e|^2}}, \tag{3}$$

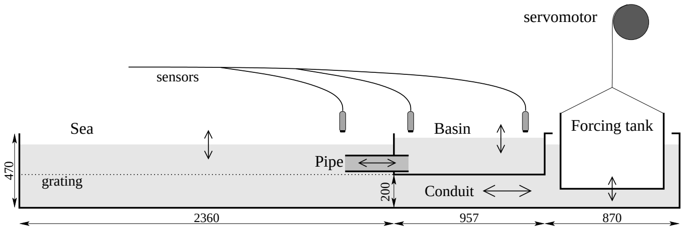
which can be used to find the phase lag $\varphi \in (-\pi, 0)$ as well, from $\varphi = \arccos((1-(\frac{\omega}{\omega_0})^2)\,\frac{|\alpha|}{|\alpha_e|})$.

III. EXPERIMENTAL SETUP

A sketch of the experimental setup is shown in Fig. 2. The area of interest is the "basin" of 0.916 m^2. Instead of a channel, a completely submerged pipe is used to connect the basin to the forcing tide at "sea". The experiments presented in this paper were performed using a circular pipe with a length of 441 mm and 76.4 mm diameter. Without the "added mass effect" (i.e. the contribution of water motion outside the pipe to the inertia of the flow) increasing the effective pipe length, the Helmholtz frequency is expected to be $\omega_0 = 0.33$ rad s^{-1} $= 53 \cdot 10^{-3}$ Hz (18.8 s period). A forcing tank is immersed in the water and is lifted and lowered by a properly counterbalanced servomotor. The control signal can be generated by the computer. In principle the forcing signal may consist of many frequency components, but harmonic forcing only is considered in this paper. The water expelled by the forcing tank, moves through the conduit underneath the basin area towards the sea. A grating was inserted in order to help the waterlevel rise at sea to be uniform, but it did not prevent the occurrence of a standing wave: depending on the forcing frequency amplitudes are higher at the far end of the sea. This does not seem to influence the measurements though. Acoustic sensors are used for nonintrusive measurements of waterlevel elevation. They are located above the tank and emit an acoustic signal at 300 kHz down to the water surface and



(a) Top view



(b) Side view

FIG. 2: Sketch of the experimental setup, dimensions in mm are indicated. The forcing tank is set into motion by a servomotor. The water expelled by the tank moves through the conduit underneath the basin area to the "sea". The tidal signal thus generated at sea, propagates into the basin through a completely submerged pipe. The water level elevation is measured by acoustic sensors at the indicated positions. Because the tide in the basin is nearly uniform, the measurements for the sensors in the basin are virtually the same.

measure the return time of the reflected signal, like an echo sounder. Their resolution is 0.36 mm, the standard deviation of the noise is similar. Significant outliers of some centimeters occur and are dismissed from the analysis.

No dissipators have been installed at the sidewalls of the sea to reduce reflections. Raichlen and Ippen[47] warn that this has a dramatic effect on the response curves. They define the amplification factor between the tidal response amplitude in the basin and the amplitude at the seaward entrance *if the basin were closed.* Indeed, radiation damping effects changing the actual tidal signal at the seaward entrance, are quite different in a finite reflective sea than in a semi-infinite reflectionless sea. We however choose to correct the motion of the forcing tank such that the amplitude at the seaward entrance is the same for all frequencies and define the amplification factor between the response amplitude in the basin and the amplitude at sea *with the basin connected to it.* This effectively boils down to eliminating the effect of radiation damping and circumvents the problems mentioned by Raichlen and Ippen[47].

IV. RESULTS

A sample time series from our measurements is shown in Fig. 3. This measurement was performed at a forcing frequency $\omega = 0.0360$ Hz $= 0.226$ rad s^{-1} and forcing amplitude approximately 2 mm, at the resonance peak under these conditions (see Fig. 4). Measurements are indicated by dots, plusses and crosses, outliers by open circles, squares and diamonds. Amplification and phase lag between the tide at sea and in the basin are clearly visible. Note that the signals of both sensors inside the basin are virtually the same, so the assumption of uniform tide in the basin is satisfied indeed. Harmonic analysis has been used to fit the measurements to a sum of sinusoids at the forcing frequency and significant overtides, if any. The harmonic fits are shown in Fig. 3 as well.

From the amplitudes found this way, the amplification factor and phase lag of the forcing frequency component are calculated. By plotting them as a function of forcing frequency, the response curves are determined, see Fig. 4, for three different forcing amplitudes $\alpha_e \approx 1, 2, 5$ mm. The theory according to (2) has been fitted to the measurements minimizing the least square error in the complex α/α_e-plane. Because the effective length differs from the actual pipe length due to the added mass effect it has to be determined empirically. The same holds good for the head loss parameter f. Therefore, ω_0 and ν_0 are used as fitting parameters. The curves shown in Fig. 4 are for the best fit with $\omega_0 = 50.5 \pm 0.9 \cdot 10^{-3}$ Hz $=$ 0.317 ± 0.006 rad s^{-1}, $\nu_0 = 374 \pm 9$ m^{-1}. So the effective pipe length $L_{eff} = 48.9 \pm 1.7$ cm is 4.8 cm longer than the actual length. Furthermore, $f = 0.97 \pm 0.02$ has the same order of magnitude as Ito[42] found for motion on real geophysical scale ($f \approx 1.5$). Hence our experiments give validation to his numerical/empirical value.

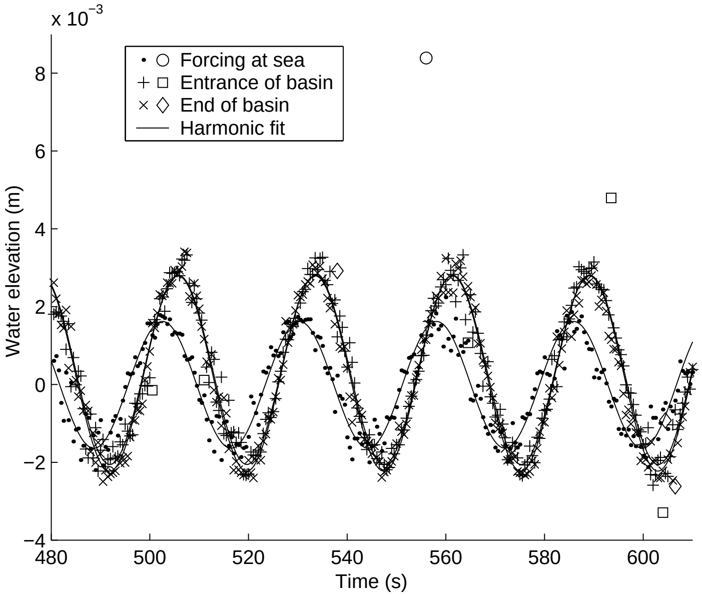
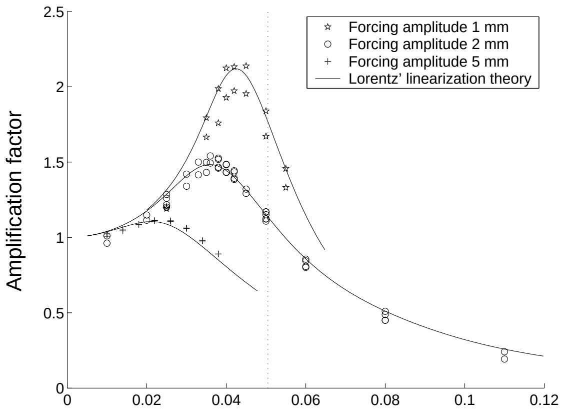
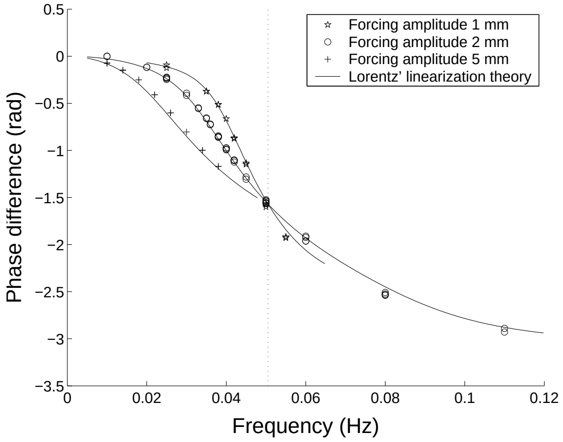


FIG. 3: Sample time series of measurements at forcing frequency 0.0360 Hz and forcing amplitude approximately 2 mm. Dots and open circles indicate measurements and outliers from the sensor at sea, plusses, open squares, crosses and open diamonds indicate measurements and outliers at the entrance and end of the basin respectively. Solid curves show harmonic fits to the respective time series. The two curves inside the basin are hardly distinguishable, consistent with a Helmholtz mode of uniform basin tide.

For a linear system there would be no dependence on the forcing amplitude. Instead the quality of the resonator appears to be higher for low amplitudes, due to the fact that effective friction decreases with decreasing amplitude, as was already noted qualitatively by Bowers[44] and De Girolamo[45]. Note that quadratic friction not only causes the response curves to be different for different forcing amplitudes but also changes the shape of the curves, because the effective linear friction coefficient $r \sim \omega\alpha$ is not constant along the individual curves either. Although this effect is hardly visible at first sight it causes the curves to drop off slightly steeper to the right of the peak frequency. A conspicuous feature of the phase lag curves is the fact that the phase lag is $-\pi/2$ for $\omega = \omega_0$ irrespective of forcing amplitude. It is immediately clear from (2) that this is predicted by the theory as well.



(a) Amplification factor



(b) Phase difference

FIG. 4: Response curves: amplification factor and phase lag between the tide at sea and in the basin as a function of forcing frequency. Three series of measurements with forcing amplitudes $\alpha_e \approx 1, 2, 5$ mm are shown, for each of which results for two sensors in the basin have been obtained. Slight amplitude differences between both basin sensors can be noted, in particular at 1 mm forcing amplitude. Solid curves show the fit to the theory (3). The Helmholtz frequency $\omega_0 = 0.0505$ Hz is indicated by a dotted vertical line. Note that one fit is used to describe all three curves. Apparently the decrease of the resonator's quality is adequately described by Lorentz' linearization theory.

V. CONCLUSION

Although the theoretical foundations of Lorentz' linearization method were already put forward in 1922[33], no quantitative high-accuracy experimental validation is found in literature yet. The theory has been developed further[29,35,36,41], comparison with observations have been made[42], but laboratory experiments have been restricted to linear theory mainly[11–15,47]. Experimental papers in which the effect of nonlinear friction on the response curves has been measured, did not provide an explicit quantitative comparison with theory, mainly because the focus was on another topic[43–45]. The experiments discussed in this paper give a clear validation of Lorentz' linearization principle to describe the influence of quadratic friction on the response curves for the Helmholtz mode in an almost enclosed basin. A good fit between theory and experiment was found.

Further research will be focussed on improving the quality of the resonator and measuring nonlinear effects from non-uniform hypsometry[8,25,26] or continuity/advection[23,27]. The first goal may be achieved by smoothing the pipe ends in order to reduce flow separation or decreasing the basin area, hence the tidal prism. For the second goal, artificial topography can be introduced into the basin mimicking intertidal flats causing the basin area to depend on the waterlevel. According to the theory this may lead to bending of the response curve, multiple equilibria and chaotic behaviour of tidal amplitudes, possibly explaining observational reports of drifting harmonic "constants"[19,20] and chaotic tidal records[21,22].

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