

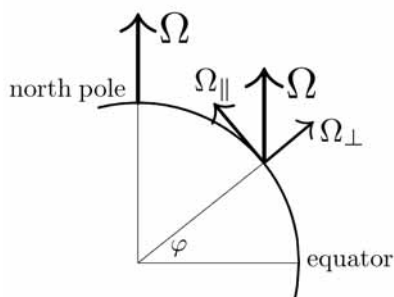
# The Coriolis force revisited

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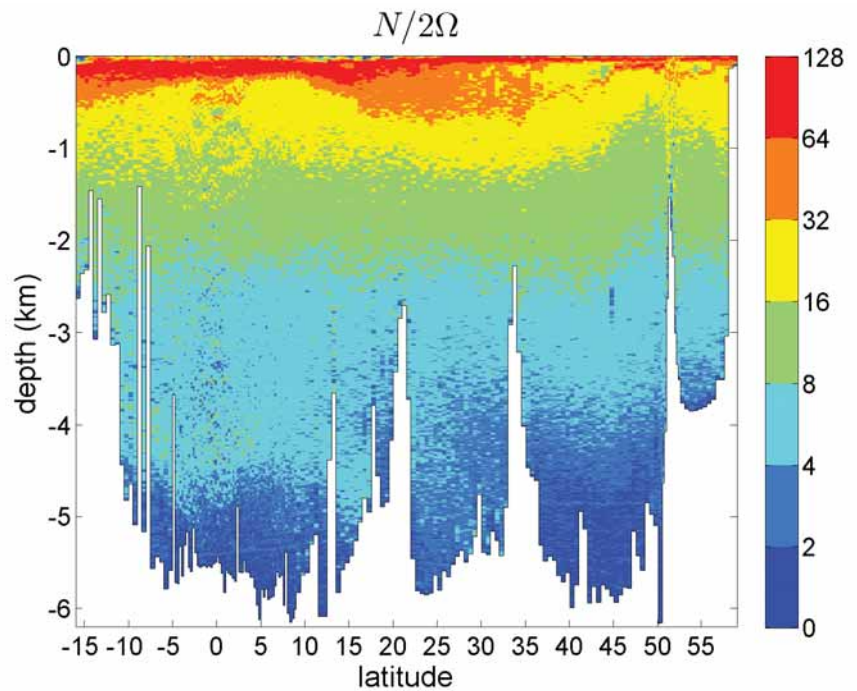
In the oceans and seas, the Earth's diurnal rotation exerts a strong influence on the way in which currents flow and waves propagate. This effect comes into play via the so-called Coriolis force, which tends to deflect moving parcels of water to the right in the Northern Hemisphere, and to the left in the Southern Hemisphere. In recent years, an often ignored aspect of the Coriolis force has elicited debate and undergone reappraisal in the communities of physical oceanographers, meteorologists and (since the same issue is at stake in rotating stars) astrophysicists.

In principle, the equations of motion contain four terms that act as a deflecting "Coriolis" force. Of these, two are proportional to the sine of latitude (and hence become stronger towards the poles), and the other two are proportional to the cosine of latitude (and become stronger towards the equator). The sine terms act purely in the horizontal geographical plane, i.e. the plane spanned by the west-east and south-north axes; these are the terms responsible for the rightward deflection in the Northern Hemisphere. The cosine terms, on the other hand, involve the vertical direction because they act in the plane spanned by the radial (i.e. vertical) and west-east axes. Specifically, this means that eastward motion induces an upward acceleration, while downward moving parcels undergo a deflection to the east.

In large-scale quasi horizontal flows, the cosine terms are relatively unimportant, and can plausibly be neglected. This is known as the "Traditional Approximation". For over two centuries, it has been applied almost universally in



A meridional slice of the Earth, through its axis of rotation. Along this axis lies the rotation vector  $\Omega$ . Decomposition of this vector at latitude  $\phi$  yields a component  $\Omega_{\parallel}$ , pointing northward, and a component  $\Omega_{\perp}$ , pointing radially upward. The former is proportional to the cosine of latitude; the latter, to the sine. Under the Traditional Approximation, the component  $\Omega_{\parallel}$  is neglected, and only  $\Omega_{\perp}$  is taken into account.



Relative strength of stratification,  $N/2\Omega$ , derived from temperature and salinity profiles in the Pacific Ocean for a south-north section near 179° E (WOCE section P14 from the Fiji Islands to the Bering Sea). Note the logarithmic color scaling. In the deepest layers (dark blue), the ratio  $N/2\Omega$  is fairly low; here "non-traditional" effects (i.e. those due to the cosine terms) can be expected to significantly alter internal wave dynamics.

studies on geophysical fluid dynamics. In recent years, however, the general validity of this approximation has been called into question. Researchers have found from modeling studies that deep convection, Ekman layers and equatorial flows are noticeably different if the Traditional Approximation is not made. In the present study, the effect on internal waves is examined.

A major factor in restricting vertical movements (and hence in reducing the effect of the cosine terms) is the vertical stratification of density, a measure of which is the buoyancy frequency  $N$ .

Scaled with  $2\Omega$ , where  $\Omega$  is the Earth's angular velocity, one has a measure of the relative importance of stratification. In practical terms, one can say that the Traditional Approximation is valid whenever  $N/2\Omega$  is much larger than one. This condition is fulfilled within the seasonal and permanent pycnoclines, but certainly not in the deepest layers of the ocean, and neither in the upper mixed layer. In other words, outside of the pycnoclines, the Traditional Approximation needs to be abandoned when one carries out modeling studies on, for example, internal waves.

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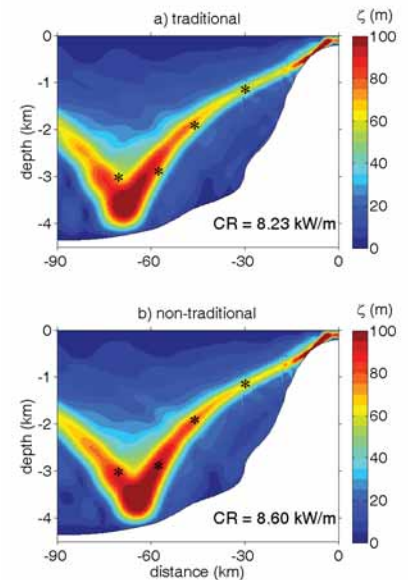
Internal waves are a ubiquitous phenomenon in the ocean. They occur over a wide range of frequencies, but most of their energy lies at the lowest part of the spectrum, specifically near the inertial (i.e.  $f=2\Omega_{\perp}$ ) and semidiurnal tidal frequencies; in the former case we speak of near-inertial waves; in the latter, of internal tides.

Very weak stratification is found in convective layers, which occur, among other areas, in the deep Mediterranean Sea. In such layers the effect of the cosine terms is expected to be profound. This is confirmed by observations. Under the Traditional Approximation, theory predicts that the horizontal current velocity field of near-inertial waves will be circularly polarized, and that such waves cannot propagate freely in homogeneous, convective layers. This, however, is not what is actually observed in the convective layer; instead, one finds that the velocity field takes the form of an ellipse, as predicted by "non-traditional" theory. In the more strongly stratified layer that lies above the convective one, the velocity field is near-circular, and here the effect of making the Traditional Approximation is slight.

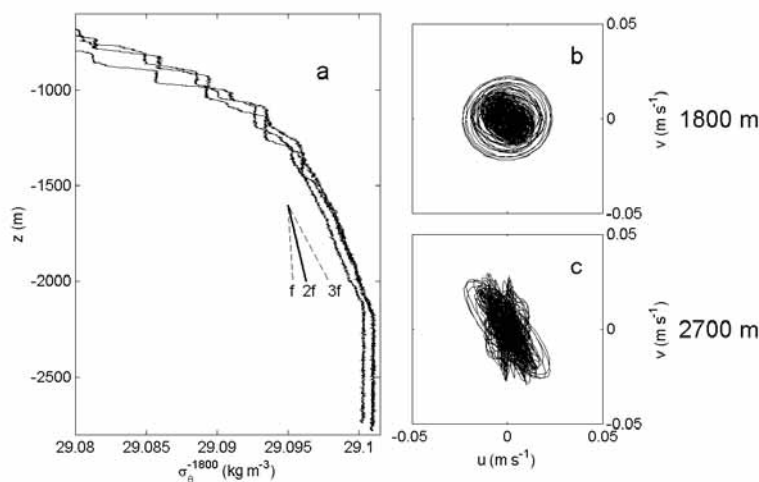
Internal tides are generated by tidal flow over bottom topography; important source regions are the mid-ocean ridge and the continental slope. Internal tides

generated near the shelf break propagate downward into the deep ocean in a beam-like fashion, as has been observed, for example, in the Bay of Biscay. Internal-tide generation models can be used to predict the trajectory of the beam as well as the associated energy flux. In recent years, the energetics of internal tides has been one of the key issues in physical oceanography because estimates of the energy fluxes are essential for determining how much energy is available for mixing in the (deep) ocean, which, in turn, is a key factor in the global overturning circulation. In the present context, the question is how the Traditional Approximation affects the direction of the beam (especially in the deep weakly stratified layers) and the magnitude of the energy flux. In order to determine this, one model calculation was carried out with, and one without this approximation. It turns out that in the latter case, the trajectory of the beam follows observations more closely; moreover, the energy flux is increased by about 5%.

In conclusion, both numerical model studies and observations indicate that the hitherto neglected effect of the cosine terms can significantly alter the dynamics of internal waves; in these cases, the Traditional Approximation should be abandoned.



Amplitudes of isopycnal excursions (in meters) from a linear internal-tide generation model, using observed stratification and topography (Bay of Biscay). In (a) under the Traditional Approximation; in (b) without the Traditional Approximation. In both plots, the asterisks show at what depth maximum excursions occur according to in-situ observations. The non-traditional model run gives a better correspondence with these observations, the downward beam originating from the shelf break being steeper, and the bottom-reflected one being less steep than under the Traditional Approximation. The associated energy flux, or conversion rate (CR), is indicated as well; its unit is in Watts per meter, where "meter" refers to the along-slope direction.



Observations from the Mediterranean: (a) profiles of potential density, showing a layer of very weak stratification below 2100 m. Hodographs based on near-inertial band-pass filtered current meter data (u east and v north) at (b) 1800 m and (c) 2700 m.