EQUATORIALLY TRAPPED INTERNAL WAVES

Contributors: Uwe Harlander* and Leo R. M. Maas

A fundamental advance in the understanding of tropical motions was made in the 60s of the previous century, namely, the development of the 'classical' theory of equatorially trapped waves. In contrast to freely propagating waves, equatorially trapped waves cannot leave the equatorial region. While propagating zonally, their energy travels back and forth between so called *turning latitudes* north and south of the equator. Beyond these latitudes wave intensity decays rapidly; between these latitudes the waves have a standing character in meridional direction, characterized by nodal lines. However, not all equatorially trapped waves can be described by this classical theory. Trapped *internal* waves, for example, are not properly described, and yet, as we argue, might be important for understanding the enigmatic deep equatorial undercurrents and off-equatorial countercurrents. To better understand the particular and sometimes surprising properties of trapped internal waves, let us first give a brief overview of the classical theory.

Classical theory

Starting point for the classical description is a system of waves that are standing in the vertical direction, between bottom and surface, which can be studied independently from the dynamics in the horizontal plane. Moreover, the horizontal structure is such that details of the model geometry, like the shape of the lower boundary or of the turning surfaces, do not influence the solution essentially. Therefore it is believed that this classical model, though strongly simplified, gives solutions that are relevant in reality too, even though a multitude of effects 'perturb' the classical model.

By linearizing the model equation and by 'freezing' the model coefficients to constant values, the dispersion relation for equatorially trapped waves can be obtained. This equation shows how wave frequency depends on wavelength. The dispersion relation is used to classify equatorial waves: waves with frequencies much smaller than Ω, the earth's rotation frequency, are called Rossby (or planetary) waves; waves with frequencies much larger than Ω are called Poincaré (or inertia-gravity) waves; waves connecting the low and high frequency regimes are the dispersive Yanai (or Rossby-gravity) waves, and the non-dispersive Kelvin waves. All these trapped equatorial waves are typical for the dynamics of an ocean or an atmosphere for which the vertical structure can be separated from the horizontal structure. In contrast to the well known wind waves, particle motion due to Rossby waves is mainly horizontal, in the meridional-zonal plane. The waves' restoring mechanism is conservation of angular momentum. This involves a change in spin, similar to what dancers experience when executing a pirouette upon contracting or extending their arms.

A sketch of an equatorial waveguide is shown in Fig. 1a. Simply speaking, the waves behave like marbles that move frictionless down a gutter while periodically climbing and descending the gutter's inclined sides. In the following we contrast the waves trapped in the horizontal plane to internal gravity waves that propagate not only in the meridional but also in the vertical direction which can thus be trapped in a meridional plane (see Fig. 1b). First, we consider again the simple case that allows for a decoupling of the horizontal and the vertical structure. Subsequently, we study the more

*Corresponding author: harland@nioz.nl

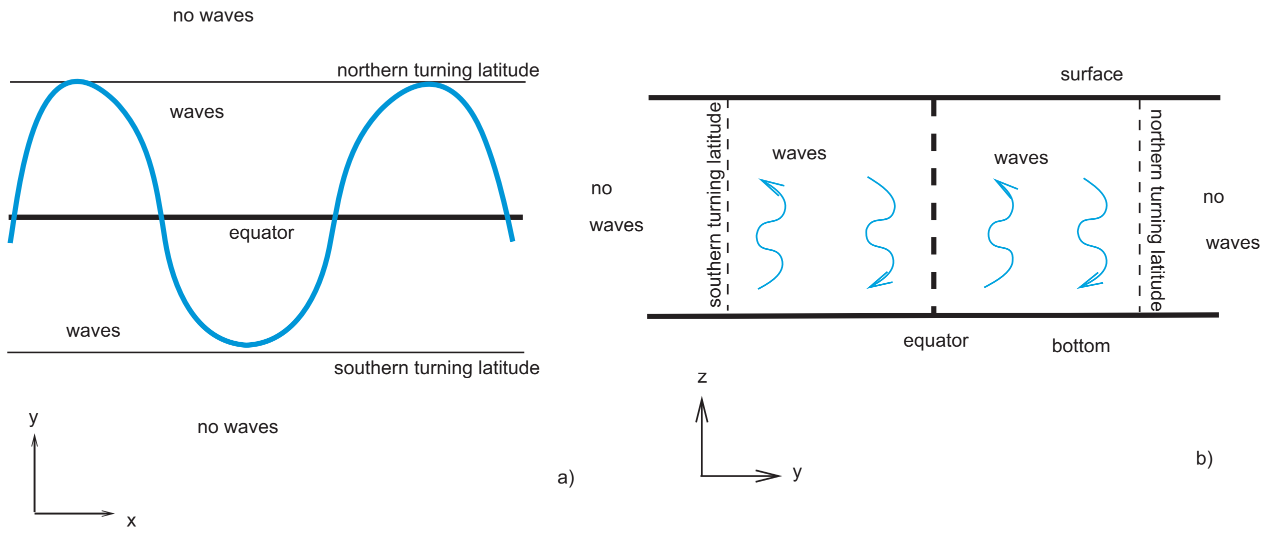

Fig.1 a) Equatorial waveguide in the horizontal x-y-plane. External gravity waves, Kelvin and Rossby waves can be trapped; b) Equatorial wave guide in the meridional y-z-plane. Internal gravity and inertial waves waves can be trapped.

realistic case for which a decoupling is not possible.

Inseperability

The main difference in the new approach is to employ the so-called Boussinesq equations that incorporate processes that have previously been neglected: vertical acceleration and nontraditional Coriolis terms. These give a more general description of oceanic and atmospheric motions. Trapped internal waves are then found to be governed by a type of equation that differs from that of the classical model. In contrast to the classical wave model, the new internal wave model does *not* possess the convenient property that small changes of the geometry or of the model coefficients lead to small changes of the solution. Trapped internal waves have several rather surprising features and we will bump into one of those below.

An example of a solution for flat and non-permeable upper and lower boundaries is shown in the vertical-meridional plane in Fig. 2. Note that we zoomed in on the Northern turning latitude (right dashed line in Fig. 1b). In Fig. 2 this (solid) turning curve is located at y=0. Displayed are contours of the streamfunction that correspond to the streamlines of the velocity field. Currents are parallel to these lines, and their intensity is inversely proportional to the distance between them. Obviously, we find a wavy pattern to the left of the turning curve (in the hyperbolic region, as mathematicians would say), and no waves to the right of the turning curve (in the elliptic region). Note that for this simple boundary geometry, the vertical structure can be considered separate from the horizontal structure, for which reason we find smooth solutions.

In Fig. 3a, 3b we plotted the spatial distribution of turbulent dissipation. In other words, the figure shows regions where kinetic energy is converted, through mixing, into heat. For the solution

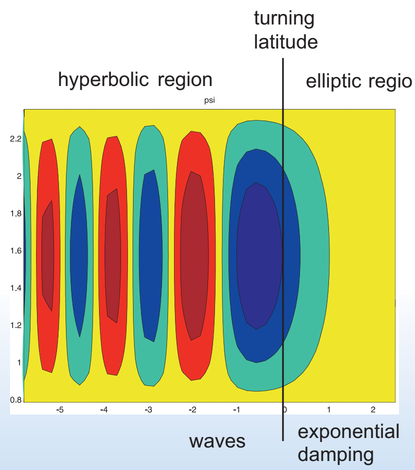

Fig. 2 Example solution of trapped internal waves close to the turning curve. The flat upper and lower boundaries allow for smooth solutions.

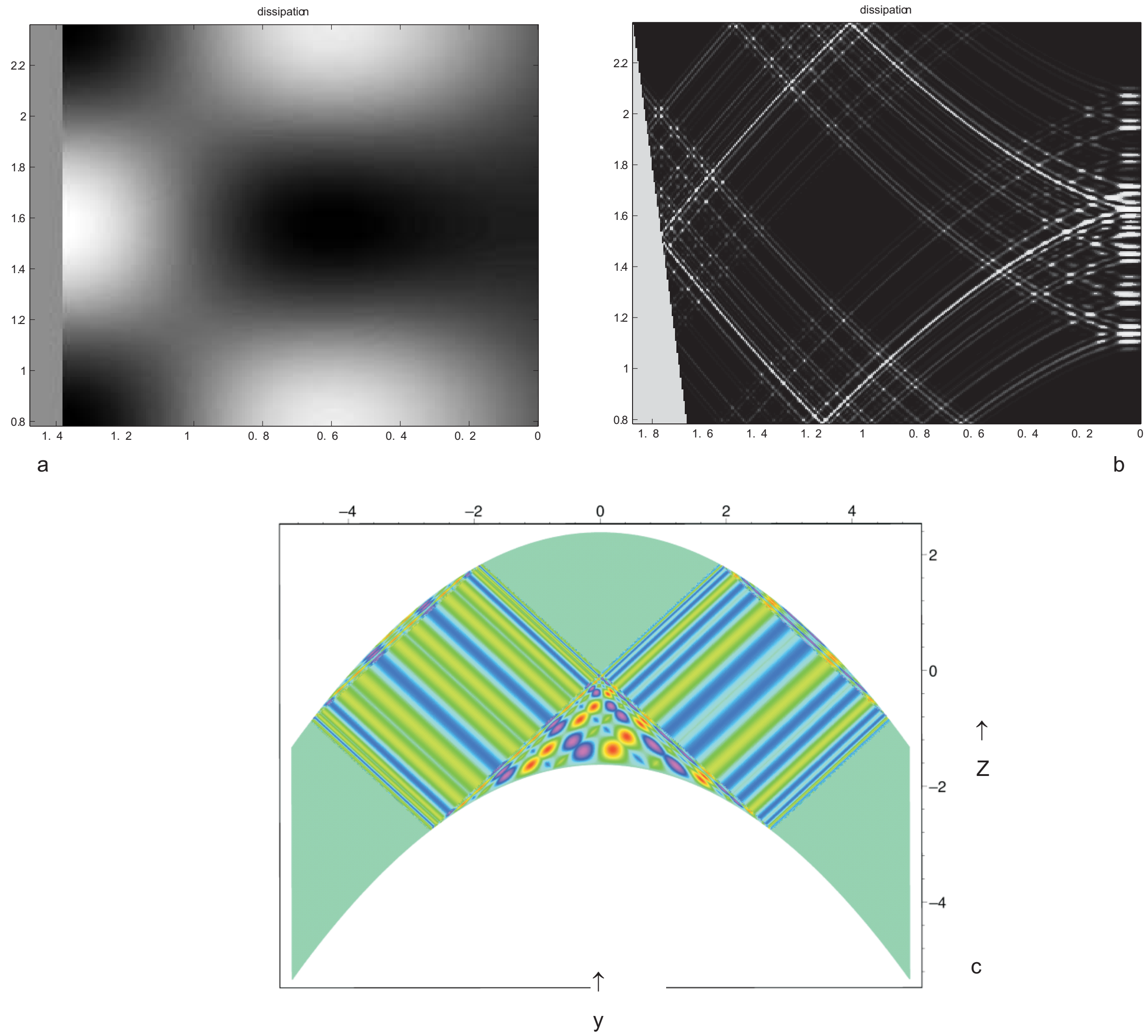

Fig. 3 a) Spatial distribution of the dissipation rate for internal waves in a rectangular box bounded to the right by a turning curve. b) Like a) but for a box with a sloping sidewall. c) Trapped inertial waves in an equatorial shell. Visible are stripes of upward (light green) and downward (blue) motion. The equator is located at y=0. Note that in a, b, and c Meridional coordinate y points to the right and vertical coordinate z upward.

shown in Fig. 2, a smooth pattern with small amplitude is found. (Note that in Fig. 3a we closed the region to the left of the turning curve by a vertical, non-permeable wall at y=-1.38). Next we would like to construct a solution for which decoupling of the horizontal and vertical structure is not possible. The most simple thing that can be done to reach this goal is to introduce a *sloping* sidewall to the left of the turning curve. Fig. 3b shows the spatial distribution of turbulent dissipation for such a case. The solution dramatically differs from the previous one: we observe a sharp boundary-detached shear layer with a very large dissipation rate. It is important to note that this result is typical for trapped internal waves, but *not* for the classical, equatorially-trapped waves considered first. In summary, under realistic circumstances that do not allow for a decoupling of the spatial coordinates, equatorially-trapped internal waves are focused towards an internal boundary layer that is called a 'wave attractor'. Such an interesting localization is not possible for the classical equatorially-trapped waves since they are governed by a different type of equation.

Shear layers in the ocean

Why should such internal shear layers be important in the oceans and where might they occur? We think that internal shear layers could enhance mixing in the deep ocean. Such a mixing is on the one hand important to keep the large-scale meridional overturning ocean circulation going, by bringing down heat adiabatically. But, on the other hand, mixing might also directly drive mean flows by mixing angular momentum and density with which the fluid is stratified. Thus internal boundary layers could connect small and large spatial scales as well as short and long time scales.

The equatorial deep ocean is a region where internal boundary layers can be expected. It should be noted that for their occurrence a sloping sidewall like in Fig. 3b is not necessary. An example of a trapped *inertial* wave (just another internal wave type) is displayed in Fig. 3c. Shown is a meridional y-z-cross-section of an equatorial shell, rotating around the horizontal y-axis. There are no walls bounding the shell in the north-south direction. Still, waves are trapped and wave motion is confined to the equatorial region. In this case, the trapping is caused by the small curvature (exaggerated in this figure) of the ocean's upper and lower boundary that leads to wave focusing. Other, extra-tropical regions where internal boundary layers could be found are elongated ocean channels, such as the Mozambique channel in the Southern Hemisphere and the Faroe-Shetland channel in the Northern Hemisphere. In the equatorial case, the deep mixing that accompanies the fine-scale structures associated with the wave attractor may present candidates for driving the ubiquitous, but enigmatic deep equatorial undercurrents and off-equatorial countercurrents for which a satisfactory explanation is presently lacking.