ROSSBY WAVES AND "RAY CHAOS"

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Contributors: Uwe Harlander and Leo R. M. Maas

To understand microscopic processes it is frequently necessary to switch from a particle point of view to a wave point of view and vice versa. For example, a laser beam can be seen as a narrow stream of particles, called photons. However, if we shine laser light through two slits and onto a wall we think of elementary waves superposing such that constructive and destructive interference lead to the well known stripy pattern on the wall. Even for macroscopic wave phenomena it is useful to attribute particle properties like path and position to waves. Throwing a stone in a pond we excite a surface wave in the form of a ring expanding in space. After a short time the leading part of the disturbance looks like a wave packet with circular crests. We can follow the packet with our eyes. Doing so we address typical local (particle) attributes to the wave: path, velocity, local wavelength, and local amplitude. In the example given above the corresponding rays consist of a bundle of straight diverging lines starting at the point where the stone has hit the pond's surface. The velocity of the wave packets (or wave groups) following the rays is called the group velocity.

Propagating wave packets can be observed both in the atmosphere and the ocean at many different length scales. The longest waves in the oceans and the atmosphere are called Rossby waves (after the Swedish-U.S. meteorologist Carl G.A. Rossby). In the atmosphere these waves are thousands of kilometers long and their crests and troughs form the well-known high- and low-pressure systems of the midlatitude weather. In the ocean, the scale is reduced by a factor ten, but similar structures, in the form of cyclonic and anticyclonic eddies betray the presence of Rossby waves. Rossby waves can propagate along complicated paths if they travel over topography or across currents. In some cases the ray path and the local wavelength and amplitude can sensitively depend on the initial conditions. Then we speak of "ray chaos."

The purpose of the study was to investigate the ray dynamics of Rossby waves in closed basins. We wanted to understand better i) trapping of Rossby waves in ocean basins and ii) the relationship between eigenmodes and periodic or chaotic wave rays. To do so we considered a particularly simple situation: an ocean channel with an almost rectangular cross section (x-z-plane). At the bottom of the channel we assume topography with a small constant slope in the x-direction. We will look upon modes constant in the along-channel y-direction. Then we can reduce the Rossby wave equation to a two-dimensional problem, tractable by ray techniques as well as more traditional methods.

Rossby waves reflect like we expect intuitively: the angle of incidence of the wave (angle between the ray and the boundary at the point where the ray intersects with the boundary) equals the angle of reflection. This simple Snell's law also holds for light waves, sound waves, and surface water waves. From the particle point of view it is immediately clear that wave solutions trapped in the deep part of the channel should occur. This is demonstrated in Fig. 1 where we can see that a wave packet following the path shown travels back and forth between one lateral boundary and a turning region in the middle of the channel. The wave packet cannot reach the lateral boundary at the shallow side of the channel. Indeed, eigenmodes (corresponding to

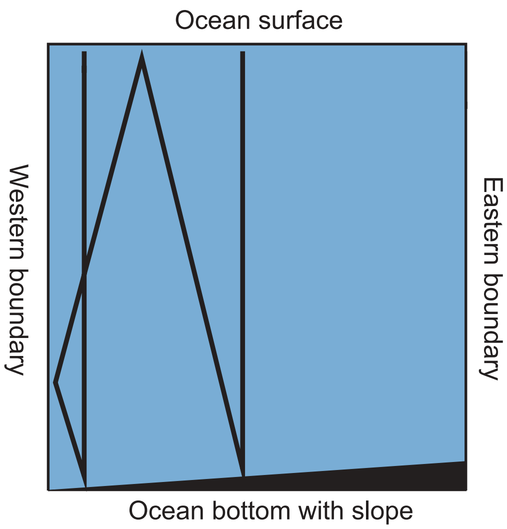

Fig. 1. Cross section (x-z-plane) of the ocean channel. The thick solid line corresponds to a ray path. Any ray hitting the lower or upper surface perpendicularly is periodic but non-closed. A wavepacket following the ray cannot reach the boundary on the right hand side. From this ray picture, trapped eigenmodes can be expected.

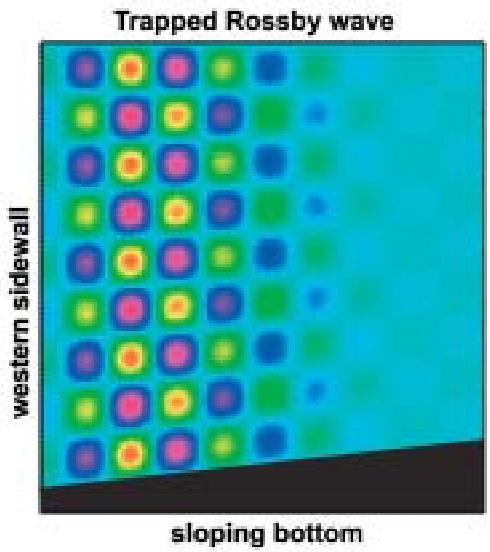

Fig. 2. Eigenmode showing wave trapping to the western (deep) part of the channel. Displayed is the vertical velocity in the x-z-plane of the ocean channel (see Fig. 1). Strong motion is indicated by red color. Only weak velocity can be found in the eastern half of the ocean. Note that the Rossby wave pattern shown is not a standing mode, but phase lines move from east to west.

the wave point of view) computed analytically show this trapping behavior too (Fig. 2). We clearly see that wave energy cannot reach the shallow part of the channel but is trapped to the western (left hand) side. The particle picture suggests that any approximation to the true geometry of the basin will have a significant impact on the solution.

For internal waves (internal gravity waves or inertial waves) it is known that a clear mode-ray correspondence exists for a 2D closed basin: eigenmodes correspond to a periodic ray pattern, i.e., all ray paths form closed curves. On the other hand, wave energy can become concentrated (focused) on a certain region in the fluid, shown by convergence of the rays to that region. This indicates the existence of a new type of solution besides common eigenmodes, called wave attractor solution. Wave attractor behavior is quite the opposite from ray chaos since all waves will --sooner or later-- propagate along the attractor, independently from the initial conditions. The question arises what can be expected for Rossby waves in a closed basin, can we find a mode-ray correspondence similar to that of internal waves?

To answer this question we tried to construct a modal solution by using a ray technique. Thereby, from the experience with internal waves, we assumed that eigenmodes should correspond to a family of periodic rays. Fig. 3 shows an eigenmode constructed by the ray technique for a flat bottom case. This solution corresponds to the exact one, available for the simple basin geometry. The mode shown has a large vertical wavelength. Such modes are not trapped and

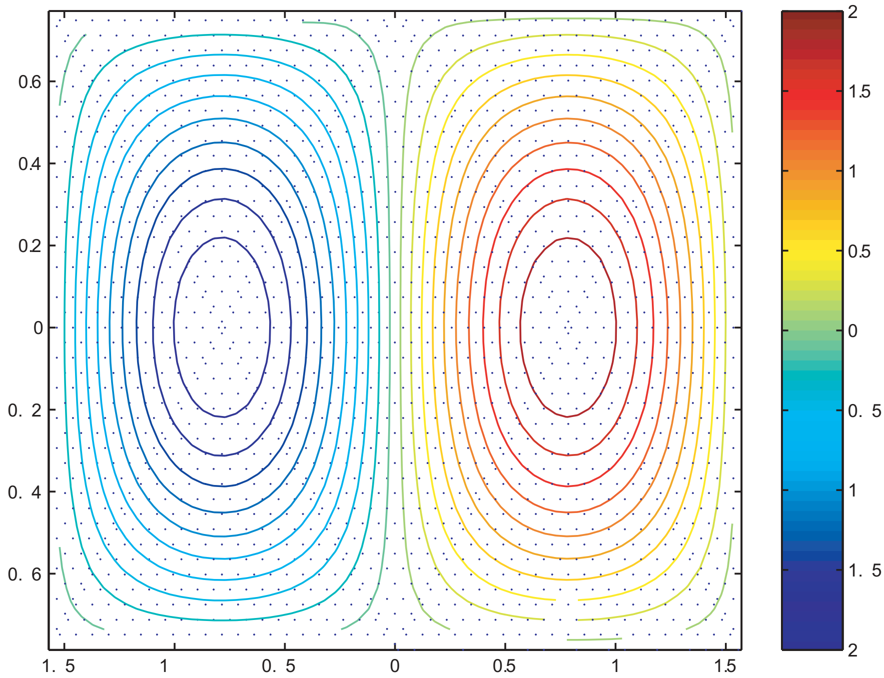

Fig. 3. Rossby eigenmode (vertical velocity) with a large vertical wavelength in a rectangular basin. The solution is obtained by using a ray technique. The blue dots indicate locations of wave ray intersections. The solution is known at such intersections and interpolated in between. Note that in contrast to the situation in Fig. 2 the mode shows no wave trapping.

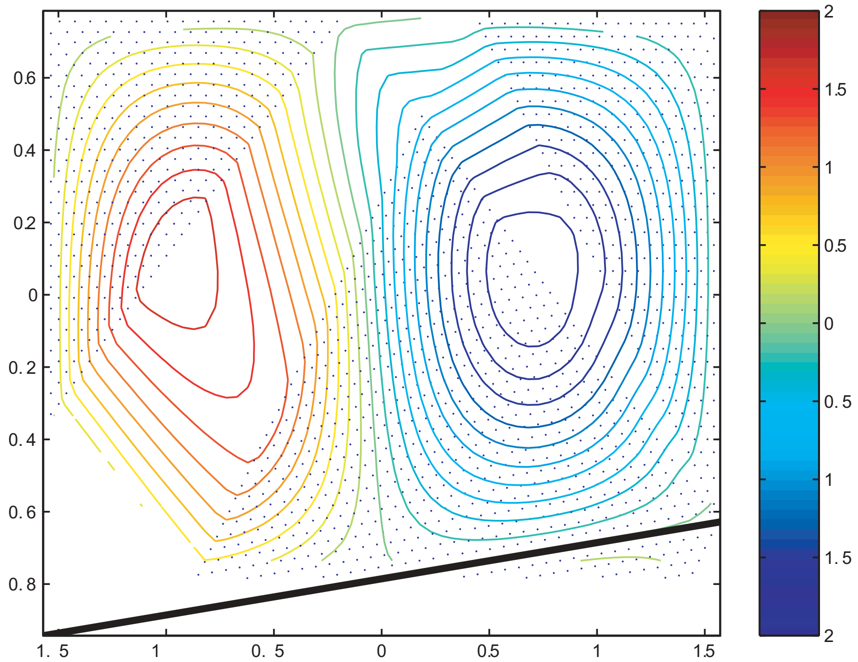

Fig. 4. The same mode as in Fig. 3 but for a basin with a sloping bottom. In the region with no blue dots (i.e., no wave ray intersections) the solution cannot be constructed by the ray technique. It looks straightforward to fill the gap by using rays starting from the lower right corner using different launch angles. However, these rays would not be periodic. Nonperiodic rays ruin the smooth structure of the mode and cannot be included in the ray/mode construction method.

corresponding rays should show reflections at both lateral boundaries, in contrast to the situation given in Fig. 1. That this is true can be seen from the fact that locations of wave ray intersections (shown in Fig. 3 by the blue dots) cover the whole domain, i.e. wave energy can reach any point in the basin.

Fig. 4 shows the same eigenmode but now for a basin with a sloping bottom. The solution agrees well with an analytical solution (obtained for small slopes, not shown here), except for the (white) fan-like region close to a ray, which connects the lower left with the lower right corner of the basin. Although the mode is not trapped, we cannot find ray intersections of closed orbits in this region, and therefore we cannot obtain a solution by using the ray technique. Fig. 5 (top) shows the closed periodic ray bounding the fan-like region in Fig. 4. The wave packet is launched at the bottom close to x = -1 with an angle of 45 degrees. All packets launched under the same angle to the right of this one will also propagate along a closed periodic path. Actually, this family of periodic orbits has been used to construct Fig. 4. In contrast, Fig. 5 (right) shows the ray path of a wave packet launched at about x = -1.2, i.e. in the fan-like region of Fig. 4. It seems to correspond to a "chaotic ray" that is filling the whole cross section without becoming closed. A field reconstruction from such chaotic rays looks very patchy and cannot be related to a smooth eigenmode. We can conclude that in contrast to the situation of internal waves, there is no simple mode/ray correspondence for Rossby waves in closed oceans. The asymmetry of the basin, introduced by a sloping bottom, leads to the fact that a single family of periodic orbits cannot cover the whole domain, as is the case for internal waves in asymmetric basins.

In summary, the study has shown that a simultaneous view of wave and particle properties can help to understand Rossby wave dynamics. E.g., geometric Rossby wave trapping can more easily be interpreted using the ray/particle picture. Eigenmode reconstruction by the suggested ray technique looks promising. However, further investigation is needed to overcome the problems discussed above.

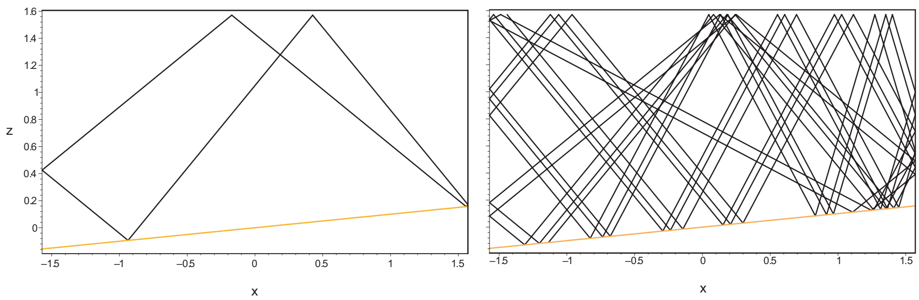

Fig. 5. Left: Periodic ray, which bounds the (white) fan-like region shown in Fig. 4. The ray is launched from the bottom at about x = -1 under an angle of 45 degrees. Bottom: "Chaotic ray", launched from the bottom at about x = -1.2 (i.e. in the fan-like region shown in Fig. 4) under the same angle. The ray corresponds to the same frequency as the one shown above but it is not closed.