THE TIDAL HELMHOLTZ RESONATOR

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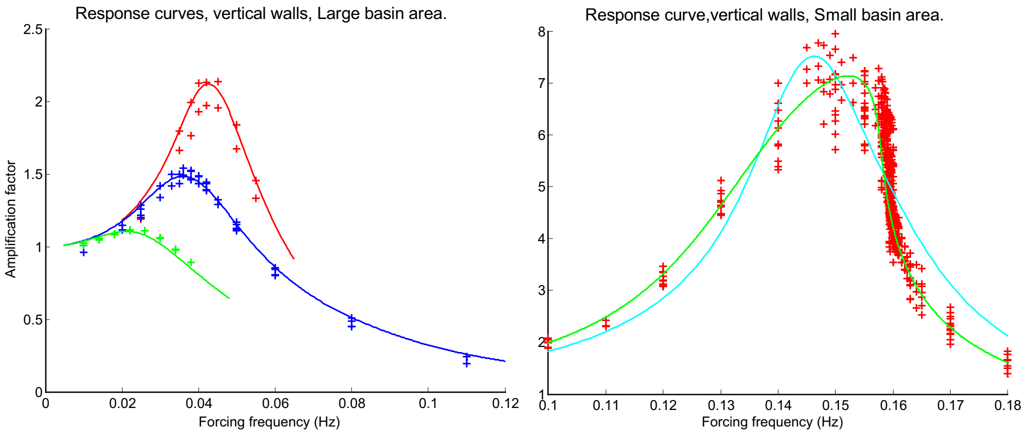
Tides are caused by the gravitational forces exerted by the sun and the moon. However, their direct forcing is not sufficient to explain the large tidal amplitudes found in coastal areas. In fact, tidal sea-level elevations generated in the oceans are amplified due to resonance of the tidal frequencies with the natural frequencies of coastal basins. A particularly simple example is the Helmholtz resonator. This is an almost enclosed basin which is connected to the sea through a narrow channel. In such a configuration a 'pumping mode' can occur: the sea-level elevation is nearly uniform in the basin's interior with the water flowing in and out through the channel while the sea-level in the basin rises and falls during the tidal cycle. Because of the lack of sea-level differences inside the basin, such a mode has a relatively low natural frequency. Therefore even small systems with lengthscales of about 20 km can be in resonance with the tidal frequency (such as the M_2 component with its period of 12h.25min).

Linear models describing the resonance phenomenon are well understood. But questions concerning nonlinear effects remain to be answered. In the model describing the Helmholtz resonator nonlinear terms arise due to (quadratic) friction and due to a non-uniform hypsometry, i.e. when the wet area of the basin is different at high water than at low water. Nonlinear friction can be dealt with elegantly using a linearization procedure developed by H.A. Lorentz. He devised this method in 1926, when supervising the scientific committee assessing the possible effects of the enclosure of the former Zuiderzee. A linear friction law is used with friction coefficient depending on the amplitude of the oscillation such that the average energy dissipation equals that if the nonlinear friction law were used. It causes friction to be more important for increasing amplitudes.

The hypsometry causes nonlinear effects because the relation between the water flux through the channel and sea-level elevations inside the basin, which acts as the restoring force of the oscillator through the resulting pressure difference, becomes nonlinear. In a mathematical model it was shown that this can have dramatic effects on the response of the tidal basin to a tidal signal at sea. Under certain circumstances the resonance curve (the amplification of the tidal amplitude in the interior of the basin compared to to tidal amplitude at sea, as a function of frequency) can be bent in such a way that 'multiple equilibria' occur: with the same tidal signal at sea, the response in the basin can be in either a choked or an amplified regime. Sudden changes between these regimes may occur and may even happen in a chaotic manner!

In order to investigate these effects, a laboratory experiment was set up in the physics department. With the tank shown in the picture, the water level in the sea can be changed by oscillating a box filled with lead at the righthand side of the tank. The water expelled by this box flows underneath the basin area to the sea, where a tidal signal is simulated. The basin is connected to the sea through a pipe, representing a narrow channel. The waterlevel changes are measured by acoustic sensors. In this way the oscillating response of the waterlevel inside the basin to the tidal signal at sea can be measured. The results so far are in good agreement with 'linear' theory when friction is very important. If the basin area is reduced, friction becomes less important and multiple equilibria are found, though under conditions not predicted by the theory. The theory thus needs to be adapted in order to describe the behavior of our small scale water tank.



Response curves:

The amplification factor between the tidal amplitude in the basin and at 'sea', as a function of frequency. The left figure is for a large basin area, with large impact of friction, for amplitudes at 'sea' of 1 mm (red), 2 mm (blue) and 6 mm (green). Theoretical curves are according to Lorentz linearization theory. The right figure is for a small basin area, with nonlinear behavior, for amplitude at sea of 1 mm (red). The blue line shows the fit with Lorentz linearization theory, the green line is with nonlinear theory.