WAVE PATTERNS DUE TO REFLECTION IN ENCLOSED ROTATING AND STRATIFIED FLUIDS

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From a physical viewpoint the sea is a density stratified, rotating fluid. This allows for a wide number of phenomena, ranging from global circulation to turbulence. A class of waves which exist in stably stratified fluids are internal waves. They propagate entirely in the interior of the fluid. The maximum amplitude is in the interior and the (free) surface hardly oscillates. The most simple example is a cup filled with water and oil, where waves propagate along the interface, but the free surface remains at rest.

For a fluid that has a stable density stratification, these waves are called internal gravity waves, since buoyancy provides the restoring force. For these waves particle motion is essentially two-dimensional. Solidly rotating fluids are stratified in angular momentum, with inertia providing a restoring force.

Internal waves in a rotating fluid are therefore called inertial waves. Due to the Coriolis force, particle motion is three-dimensional. Since internal gravity waves and inertial waves have similar properties, for a fluid that is both rotating and density stratified, they combine into an inertio-gravity wave. Such waves are widely observed on scales of tens of meters to kilometers and with periods of minutes to one day.

An essential property of the internal waves is that their direction of propagation is purely determined by the wave frequency and the strength of the stratification and/or the rotation rate. For continuous stratification (no density jumps), the waves will propagate obliquely through the fluid. When such a wave reflects, the reflected wave does not obey Snell's law, but retains its angle with respect to the direction of gravity/rotation axis, even at reflection at a sloping wall. Therefore, in a two-dimensional cross section of a channel, repeated reflection in an enclosed fluid will not lead to `chaotic' but to structured patterns. Some of the structured patterns are periodic orbits, to which all wave rays converge. They act as limit cycles and all wave energy is concentrated around them. Therefore these cycles are called wave attractors. This strong concentration of energy may induce nonlinear effects like wave-breaking resulting in mixing of the fluid, in a rotating fluid eventually leading to the development of a net flow.

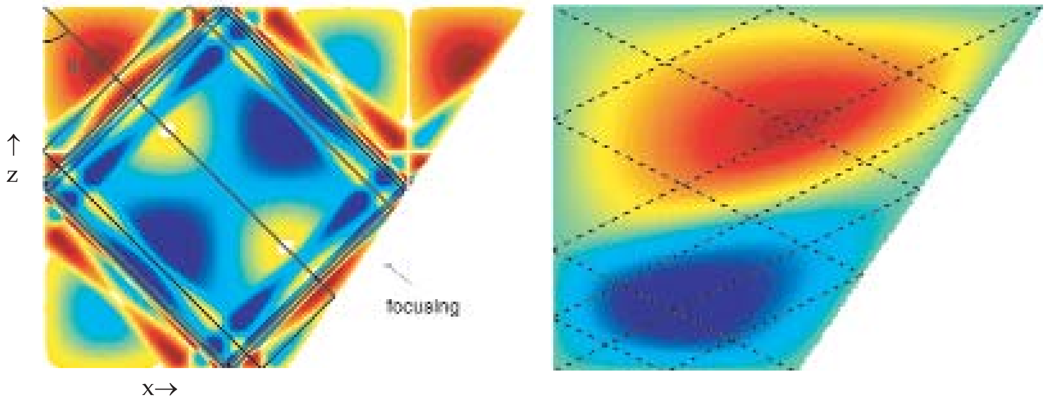
The type of equation that determines the spatial patterns of the monochromatic waves cannot be solved for arbitrary basin shapes. Numerical methods must be used, unless severe simplifications or restrictions are applied. In two dimensions one can construct solutions by describing the paths of individual wave rays. This method was used to study the existence and properties of wave attractors in a smooth basin. In three dimensions it is not possible to construct a solution this way, but alternatives are lacking. Nevertheless, description of individual rays can be used in determining the possibility of convergence towards an attractor. The formation of attractors in three dimensions has been verified in laboratory experiments. These experiments were also used to study the horizontal structure of the wave pattern.

Do wave attractors occur for smooth basins?

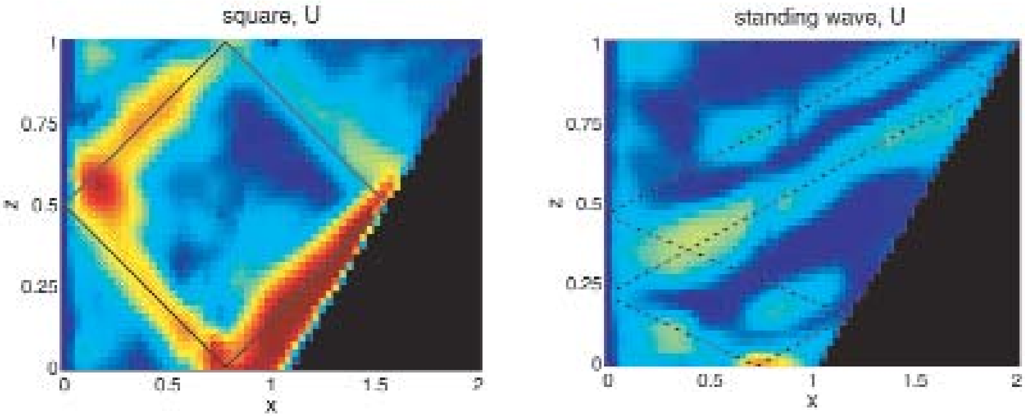
It is natural to wonder what is special about the boundaries of a fluid domain, apart from having a sloping part, to enable the formation of a two-dimensional wave attractor. Are corners necessary, or some other singularity? To answer this, a special geometry was investigated, that had completely smooth boundaries and that, depending on a parameter, varied between a circle (no attractors possible) and a symmetric triangle (where corners act as point attractors). Wave rays with different slopes were traced numerically over a large number of reflections to determine eventual convergence towards a wave attractor. For many combinations of parameter values, limit cycles were found. If the whole parameter space is plotted, bands with attractors are found that narrow when the circle is approached and which are known as `Arnol'd tongues'. When the triangle is approached the bands come closely together. In between the bands with attractors, thin regions are present where wave rays do not converge, but are shifted continuously, as well as thin regions with weak attractors of high period. Only for a single value of the direction of propagation every wave ray closes exactly onto itself and a standing wave can exist. This is because an additional symmetry exists for this direction of propagation. In the rest of the parameter space, this symmetry did play a role in the exact ordering of the tongues and their period (loosely speaking: number of reflections of the periodic orbit). The tongues were bounded by slope values for which the periodic orbit degenerated into a line, connecting critical points (points where the tangent to the bounary has the slope of the characteristic) or where two coexisting attractors merged into a single symmetric attractor.

Laboratory observations

Wave attractors and a standing wave were also investigated in the laboratory. A rectangular tank was provided with a sloping side wall over the length of the tank to make focusing



For a fixed frequency and a fixed stratification, an internal wave ray travels obliquely through the fluid with a fixed angle θ with respect to the vertical. This angle is conserved upon reflection. Reflection at a sloping wall leads to (de)focusing. In an enclosed basin, net focusing leads to the appearance of a wave attractor (left), where all wave energy accumulates. This is illustrated by the stream function. The flow is parallel to its isolines, the strength is indicated by the accumulation of isolines. When focusing is balanced by defocusing, a standing wave exists, for which all wave rays close onto themselves (right).



Experimental observations (velocity) of a wave attractor (left) and a standing wave (right). The velocities of for the standing wave are much smaller, the colour scale differs by a factor 2.
Red (blue) marks large (weak) velocities. Black solid line (left) gives location of attractor. Dashed line (right) is just one of the many periodic wave paths.

towards a wave attractor possible. This tank was filled with tap water and small neutrally buoyant particles to visualise the flow. The tank was placed on a rotating platform, of which the rotation speed was modulated slightly to generate inertial waves with the rotation frequency. By illuminating a thin sheet of fluid and recording the particle motion in time with a digital camera, two-dimensional velocity fields could be obtained (Particle Image Velocimetry). Measurements were repeated for different planes of observation, both in horizontal and in vertical planes, and for different frequencies (different structures). Two sets of experiments have been done. One in a large tank (107 cm wide, 500 cm long, 80 cm high), at the 13 m diameter rotating platform of the Coriolis Laboratory in Grenoble, France, where six different frequencies were investigated. The other in a much smaller tank (19 cm wide, 40 cm long, 19.5 cm high) on a 1 m diameter tank at the Fluid Dynamics Laboratory of the Technical University of Eindhoven. In this second set the horizontal structure could be observed better. For such a three-dimensional tank, ray tracing still predicts the appearance of wave attractors, since due to reflection and refraction at the sloping wall a wave that initially propagates in the horizontal direction can be `stuck’ in the down-channel direction, when approaching a limit cycle. This is not possible for standing wave modes. However, near the vertical front and end walls of the `channel’ a wave attractor cannot exist, there is a limitation to the value of ray theory and the experimental results must give insight.

Attractors were observed in both sets of experiments, as well as a standing wave. Patterns observed in vertical planes appear to agree well with the theoretical prediction. In horizontal planes the behaviour is quite different for the different frequencies. For the lowest frequency, the wave rays are the steepest and they are tangent to the slope. They are attracted immediately to the limit cycle and the attractor can be observed close to the vertical front wall. In the horizontal direction the phase of the particle motion propagates and the particle orbits are different in different sections. The horizontal wave length seems related to the horizontal aspect ratio of the tank. The `square' attractor, which is non-degenerate, is not clearly visible near the front wall, but is well observed around 1/4 and 3/4 of the length of the tank, and weakly or hardly visible in the middle. Although there is phase propagation along the attractor in the vertical planes, in the horizontal plane the attractor behaves like a standing wave with a nodal line at 1/2. The standing mode has relatively weak particle motion, i.e. about half of that of the attractors. It is hardly visible near the vertical walls, but it is more clearly visible towards the middle of the tank in the Eindhoven-experiments, where it behaved like the most simple standing wave mode in the horizontal direction, although in the vertical planes some phase propagation was observed. In the Grenoble-experiments there was clear standing wave behaviour in the vertical plane around 1/4 of the length The behaviour in the horizontal direction could not be well observed.

The real sea

Although internal tides (internal waves of tidal frequency) have been observed widely, the observations often do not tell much about their spatial distribution. Simple attractors as observed in the laboratory can hardly be expected, but it is worthwile to study their interaction with topography to find regions of stronger wave activity and mixing areas. In the ACSEX-project, an array of current meter moorings was deployed for 1.5 years in the Mozambique channel. These moorings were primarily set out to study the large scale flow, but since the Mozambique channel has steep topography and a reasonably strong tide, the measurements were also used to look for internal tides. Due to the different ray paths of the internal waves of different frequencies (M2,S2,K1), a combination of these internal tides will give amplitudes and phases that change with every location. Furthermore the ray paths change with changes in stratification, which occurs with the passage of large eddies in the channel. The current meters themselves change location during an eddy passage, since the strong flow causes considerable tilting of the moorings. Therefore changes in phase and amplitude are expected. The current meters were separated by several hundred meters in the vertical and tens of kilometers in the horizontal, much larger than the internal wave scale. Therefore, a two dimensional numerical model was used to predict the wave patterns and phases in the channel. This is currently used for comparison to the observations, but the large separation scales seem to render the different current meter results less conclusive with regards to the detection of wave focusing and attractors.

We conclude by stating that wave attractors, theoretically predicted for two dimensional containers, are observed to arise also in laboratory experiments in three dimensional containers. Field experiments in the Mozambique Channel have been spatially undersampled and therefore remain inconclusive with respect to the occurrence of large scale wave attractors in the natural environment. This invites for a finer sampling strategy. This work is funded by the NWO-NLS programme.